Cross-check of different techniques for two-dimensional power spectral density measurements of X-ray optics

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Outline

- *Power spectral density (PSD) from surface roughness data – definitions*
- *Motivation for 2D PSD measurement*
- *PSD measurements with three different instruments:*
  1. *Micromap interferometric microscope,*
  2. *Atomic force microscope*
  3. *X-ray Reflectivity and Scattering experimental facility*
- *Improvement of the Micromap measurements*
  1. *Estimation of 2D MTF*
  2. *Correction of the 2D PSD measurement with the Micromap*
- *Lower spatial frequency measurements with the LTP and ZYGO*
- *Conclusions*

Two Dimensional Power Spectral Density

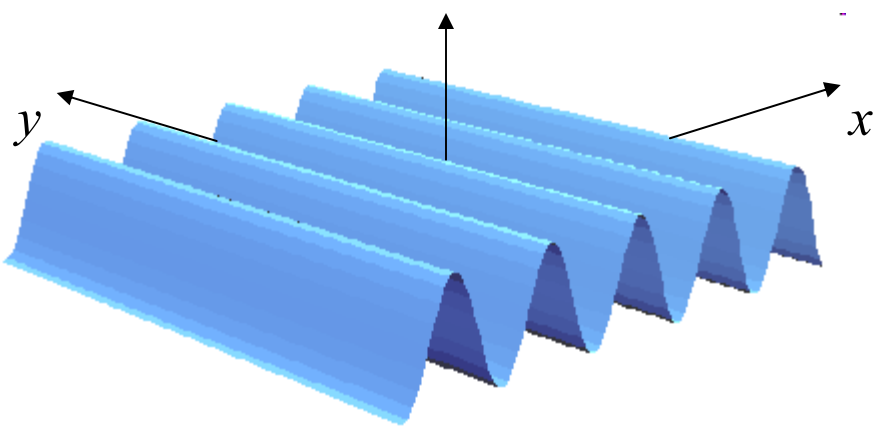
The 2D PSD function may be viewed as a Fourier decomposition of the 2D surface height distribution into harmonic basis functions:

$$S_2(u,v) = \lim_{A\to\infty} \left\langle \frac{1}{A} \left| \int_{-L_y/2}^{L_y/2} dy \int_{-L_x/2}^{L_x/2} h(x,y)\, e^{-2\pi i (u x + v y)} dx \right|^2 \right\rangle$$

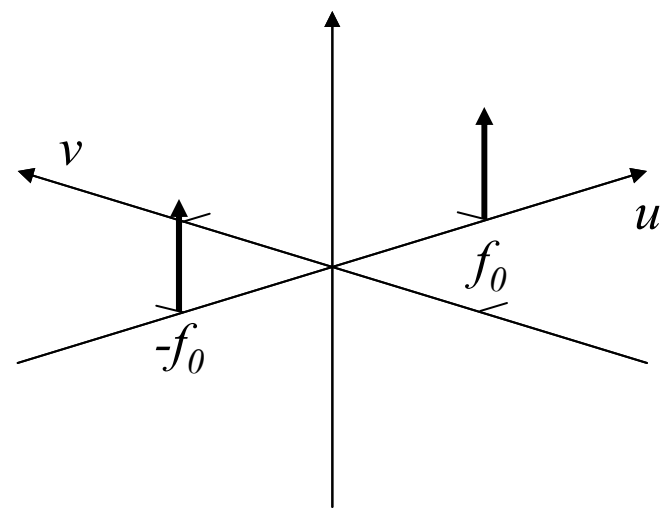
L_x and L_y are the tangential and sagittal dimensions of the measured surface region, A; u and v are the spatial frequency variables corresponding to the tangential, x, and sagittal, y, coordinates.

Cosine grating

$$h(x,y) = Cos(2\pi f_0 x)$$



$$S_2(u,v) \sim \delta(u-f_0,v) + \delta(u+f_0,v)$$



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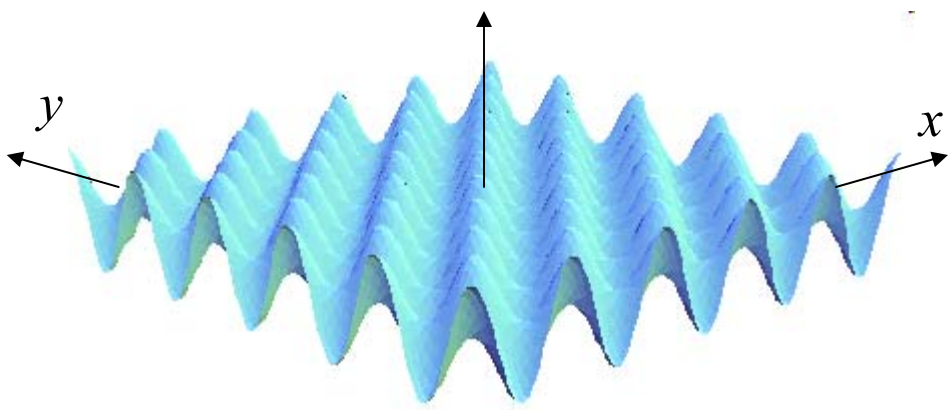
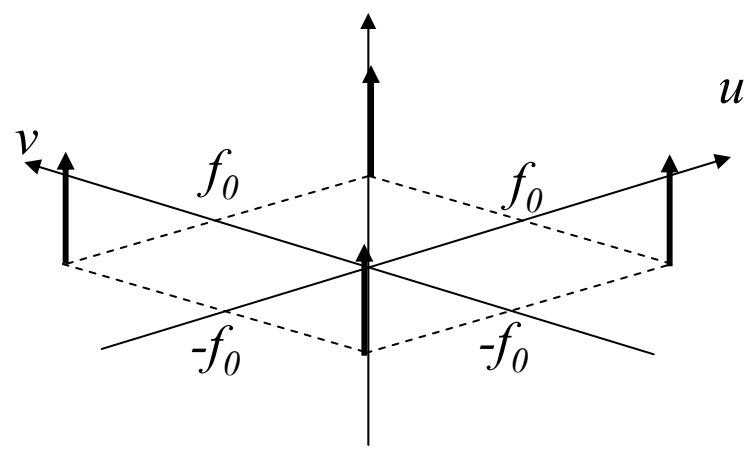
2D Cosine grating

$$S_2(u,v) \sim \delta(u-f_0, v-f_0) + \delta(u-f_0, v+f_0) + \delta(u+f_0, v-f_0) + \delta(u+f_0, v+f_0)$$

$$h(x,y) = h_0 \, Cos(2\pi f_0 x) \, Cos(2\pi f_0 y)$$





Two Dimensional Power Spectral Density

In the case of discreet measurements, 2D PSD distribution can be evaluated from

$$S_2(l,k) = M\,N\,\Delta x\,\Delta y\left|F_{l,k}\right|^2$$

$$F_{l,k} = \frac{1}{M}\sum_{m=0}^{M-1}\left[\exp\left(\frac{-2\pi i\,m\,l}{M}\right)\frac{1}{N}\sum_{n=0}^{N-1}h_{m,n}\exp\left(\frac{-2\pi i\,n\,k}{N}\right)\right]$$

M and N are the number of pixels and Δx and Δy are the pixel dimensions in the tangential and sagittal directions, respectively; $F_{l,k}$ are the elements of the Fourier transform matrix

One-sided (positive frequency only) 1D PSD distribution can be evaluated from 2D PSD:

Tangential: $$S_1(k) = 2\,g(k)\sum_l S_2(l,k)$$

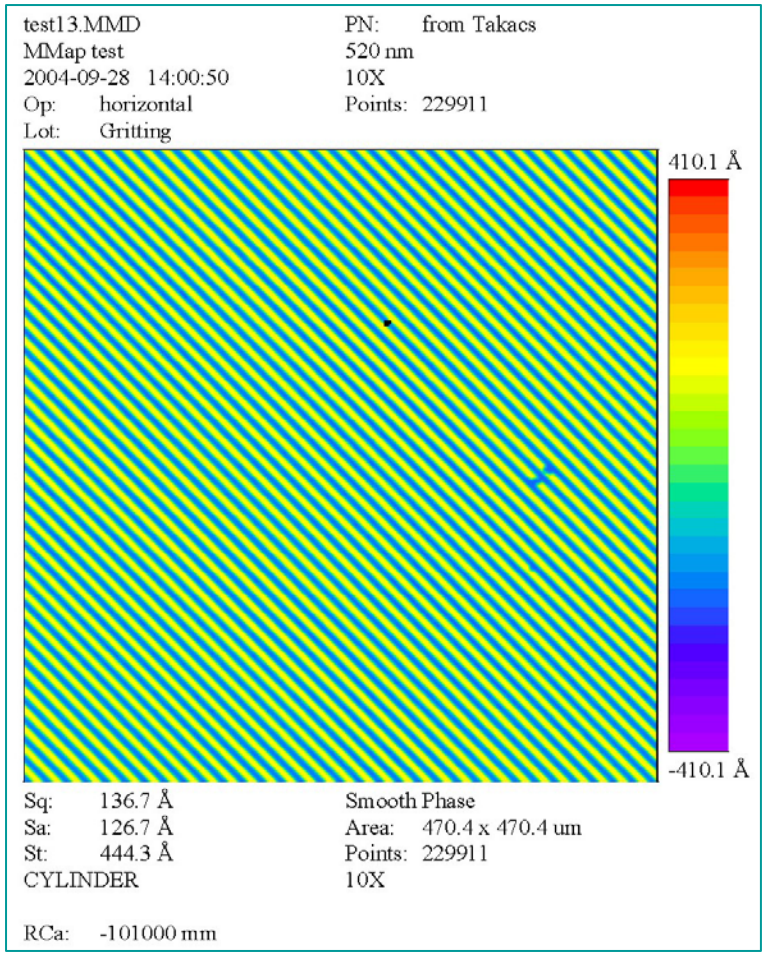
Sagittal: $$S_1(l) = 2\,g(l)\sum_k S_2(l,k)$$

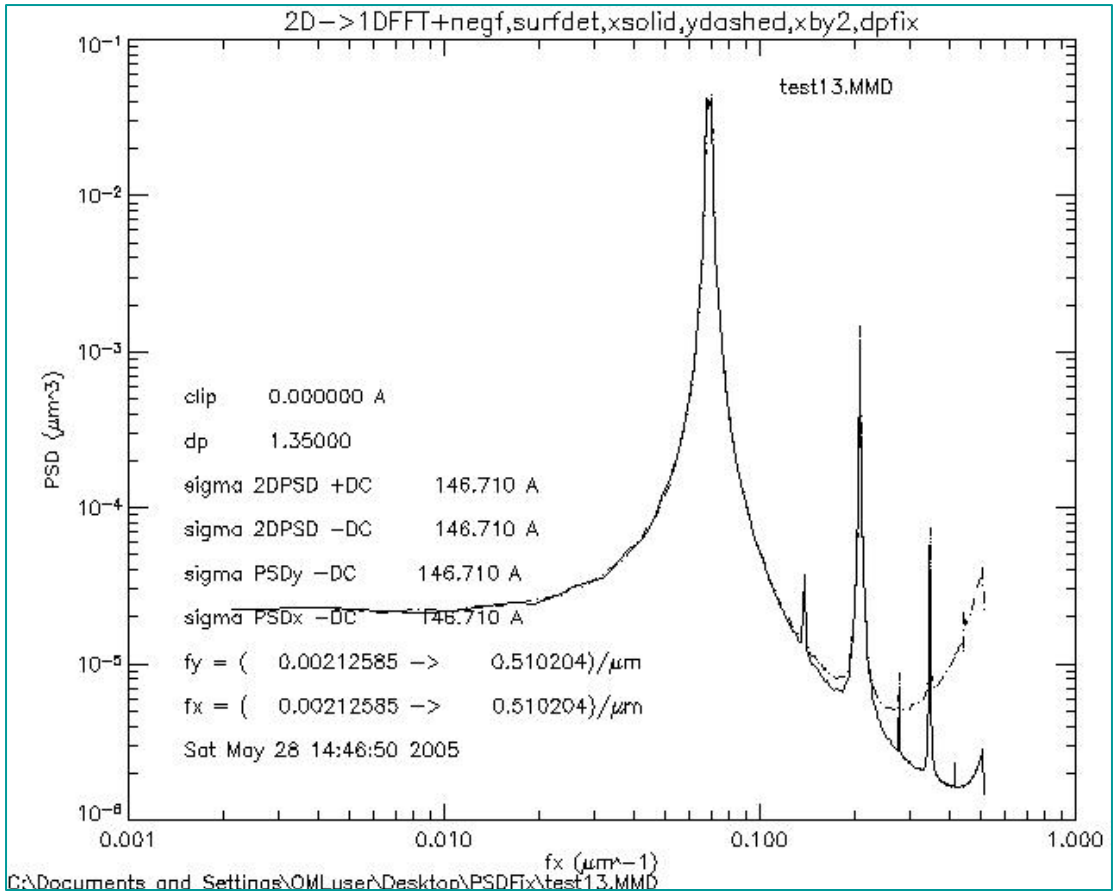
$$\begin{cases} 0 \le l \le M/2, \quad 0 \le k \le N/2, \\ g(l) = 1/2 \quad at \quad l = 0,\, M/2, \\ g(k) = 1/2 \quad at \quad k = 0,\, N/2, \quad and \\ g(l) = 1 \quad and \quad g(k) = 1 \quad otherwise \end{cases}$$

Motivation for 2D PSD measurement

→ ***PSD provides rigorous information on surface properties***

Micromap measurements with a X-ray grating with uniform groove density



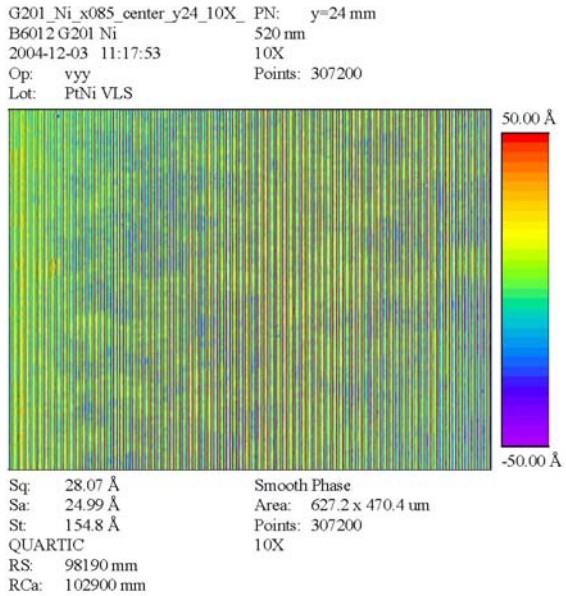


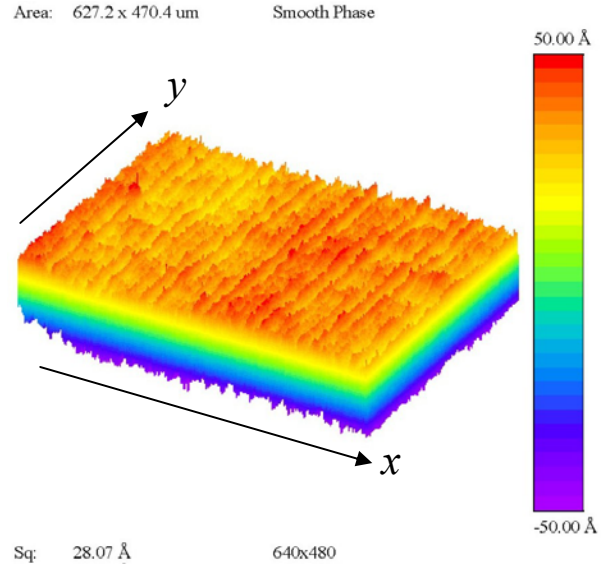
Motivation for 2D PSD measurement

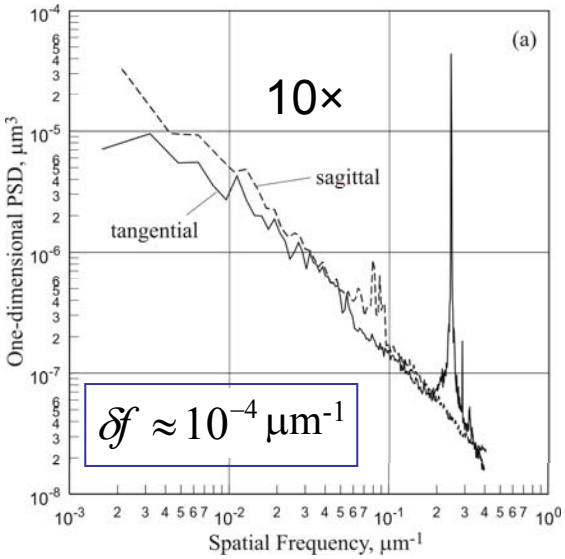
→ ***PSD provides rigorous information on surface properties***

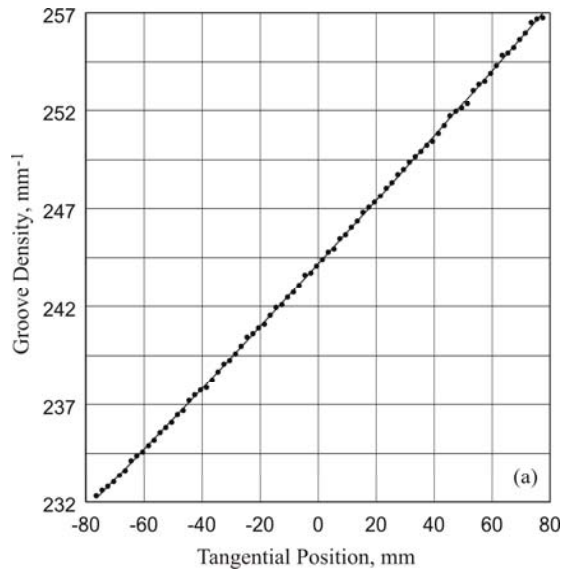
Micromap measurements with a X-ray grating with variable groove density

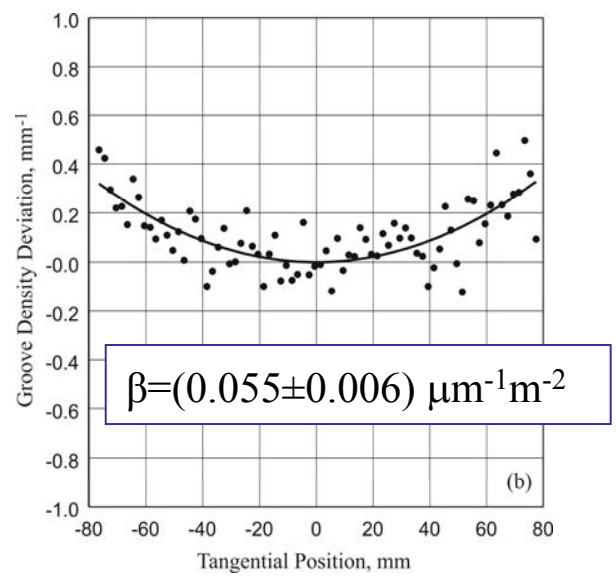
(Proceedings of SPIE 5858-10)







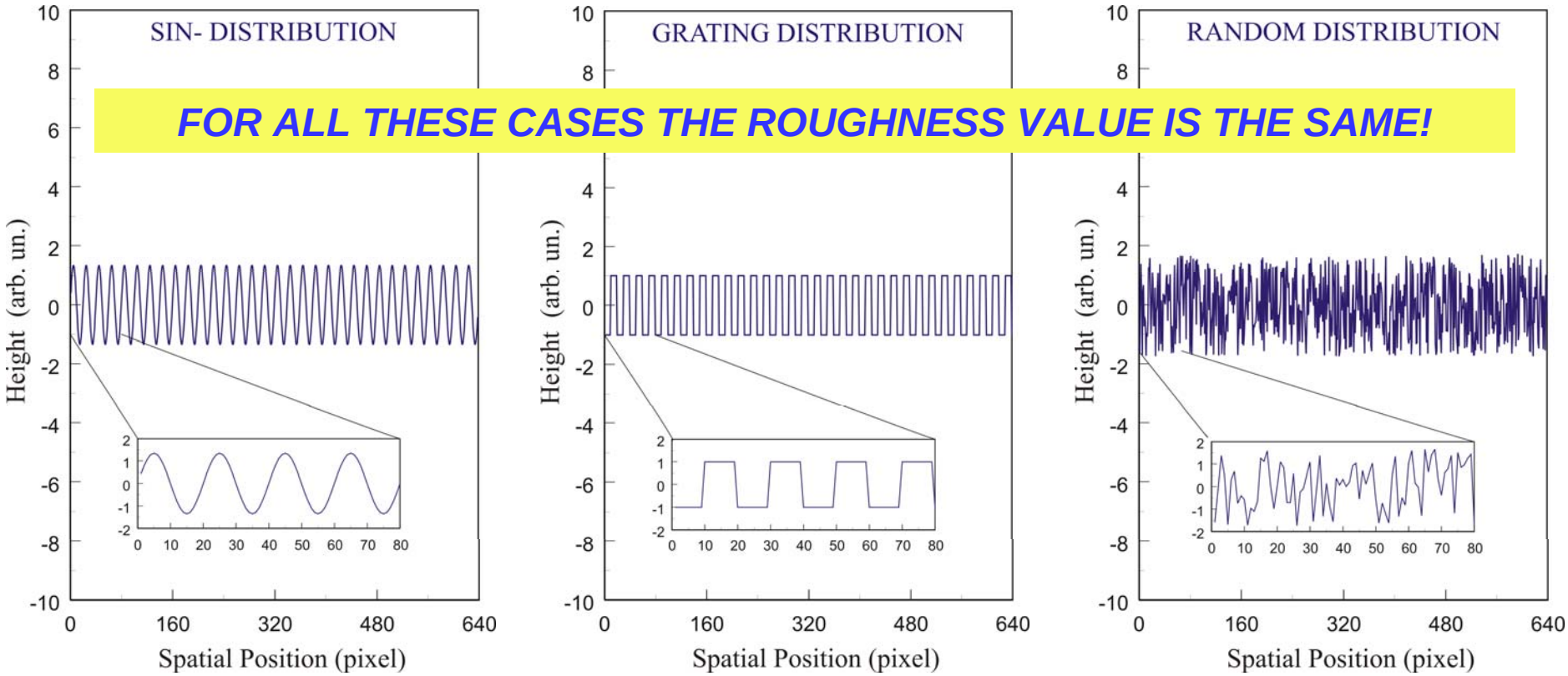




Motivation for 2D PSD measurement

→ ***PSD is based on essential information on surface properties***

Roughness (rms): $$R_{RMS} \approx \left[\frac{1}{N} \sum_{n=1}^{N} (h_n - \hat{h})^2 \right]^{1/2}$$



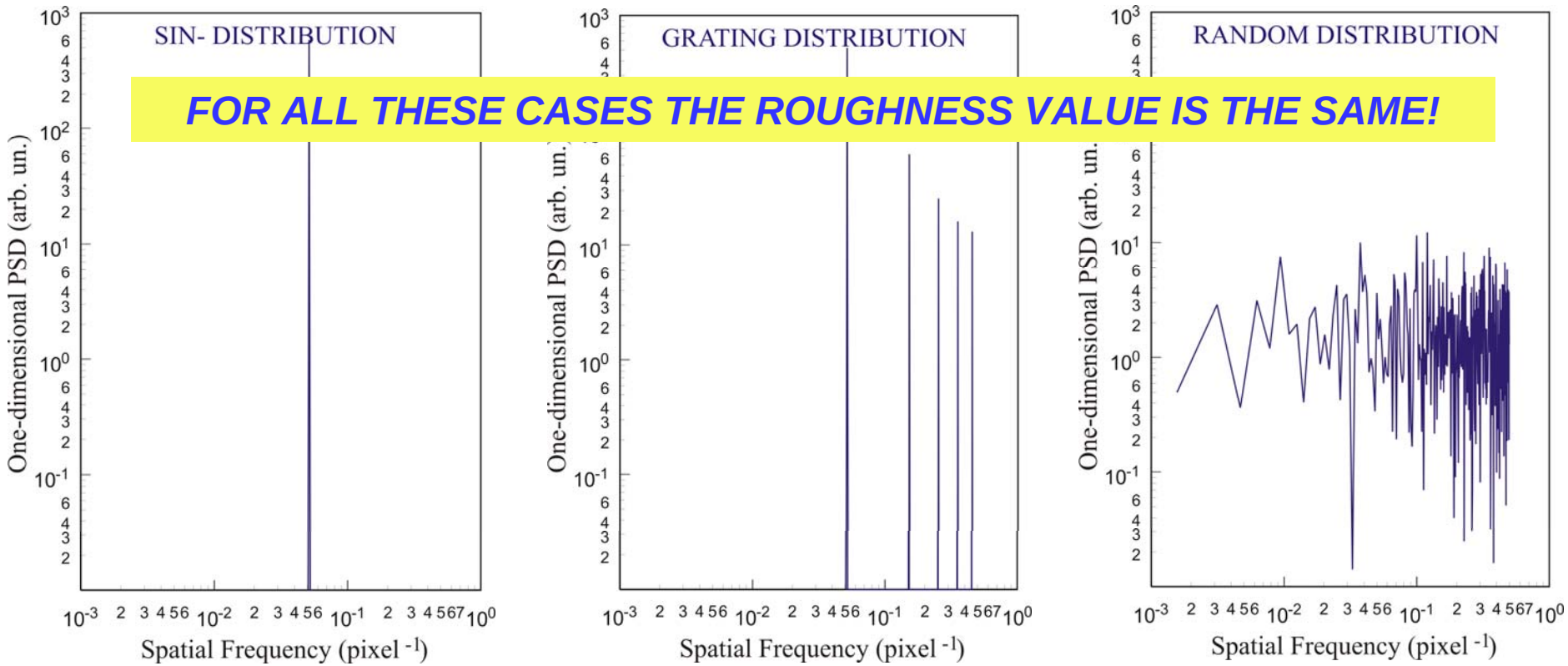
FOR ALL THESE CASES THE ROUGHNESS VALUE IS THE SAME!

$$h_M(x,y) = h_S(x,y) \otimes OTF(x,y)$$ *- convolution*

Motivation for 2D PSD measurement

→ ***PSD contains essential information on surface properties***

Roughness (rms): $$R_{RMS} \approx \left[\sum_{i=1}^{I} S_1(f_i) \Delta f_i \right]^{1/2}$$



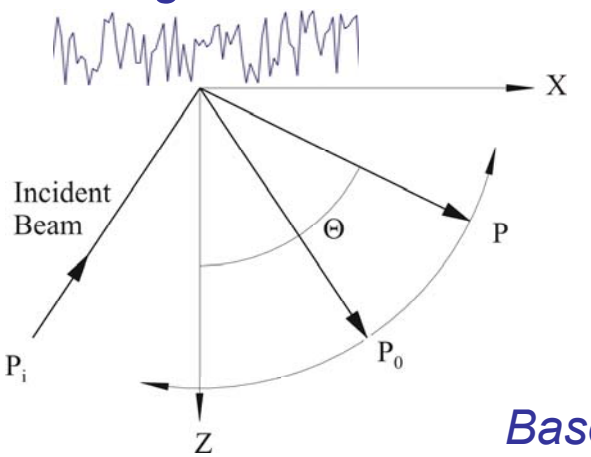
FOR ALL THESE CASES THE ROUGHNESS VALUE IS THE SAME!

$$S_M(u,v) = S_S(u,v) \cdot MTF^2(u,v)$$ *- product*

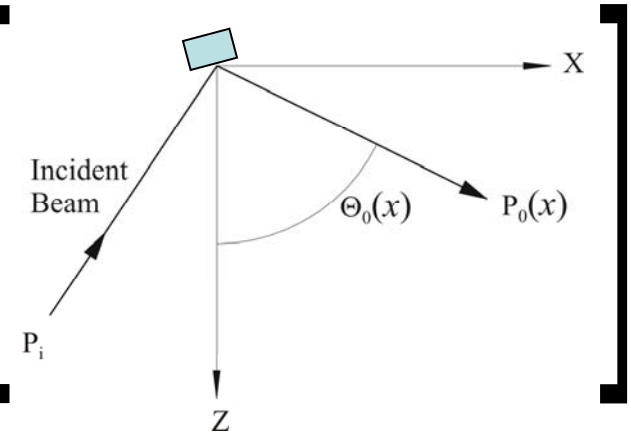
Motivation for 2D PSD measurement

→ *2D PSD allows calculation of 3D scattering of X-rays*

Scattering from a real surface



$$= \sum_{x=x_{\min}}^{x_{\max}}$$



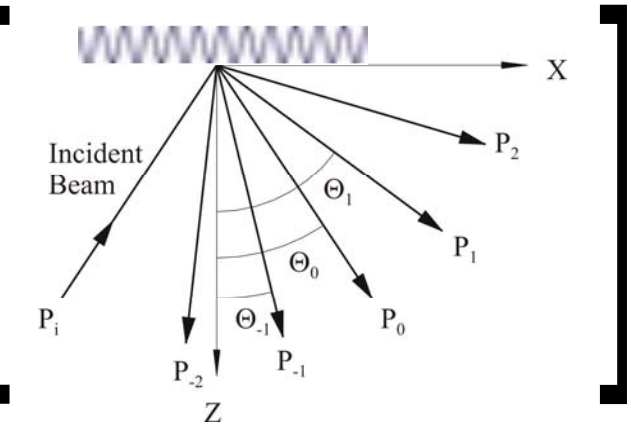
Based on correlation function parameterized with σ_{rms} and l_{cor}

$$\| $$

$$\sum_{f=f_{\min}}^{f_{\max}} \left(a\, Sin(\pi\, f\, x + \alpha) \right)$$

$$\left(\begin{matrix} \text{Fourier} \\ \text{Transformation} \end{matrix} \right)^2 \propto \text{PSD}$$

$$= \sum_{f=f_{\min}}^{f_{\max}}$$

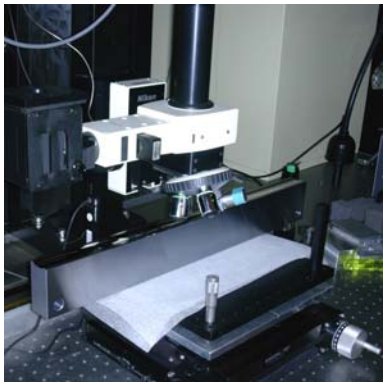
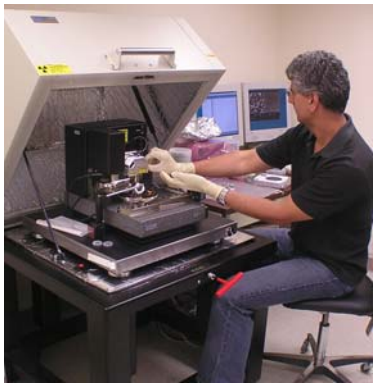


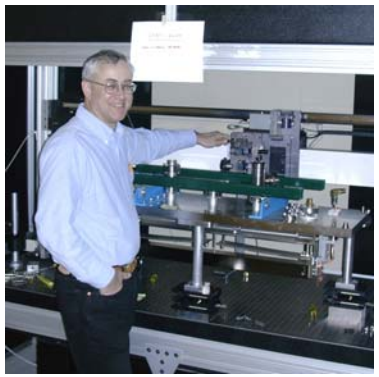
Based on PSD parameterized with $S_2(0)$ and n

LBNL X-ray optics Metrology

The metrology instrumentation

- ✓ Micromap™-570 interferometric microscope
- ✓ Digital Instruments Dimension 3100 Atomic Force Microscope
- ✓ X-ray Reflectivity and Scattering experimental facility
- ✓ Zygo™ GPI 6-in interferometer
- ✓ LTP II long trace profiler

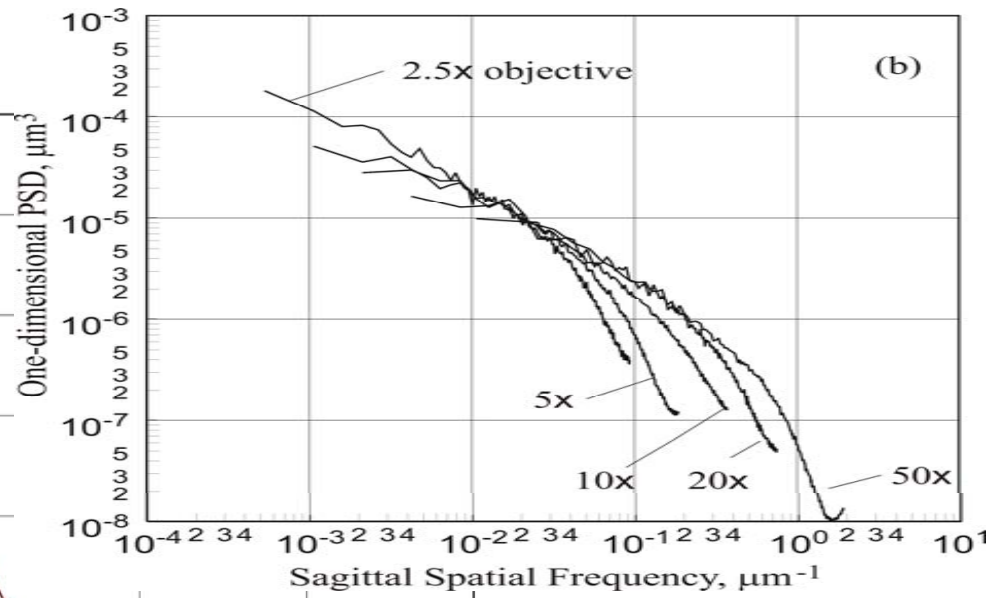


PSD measurements

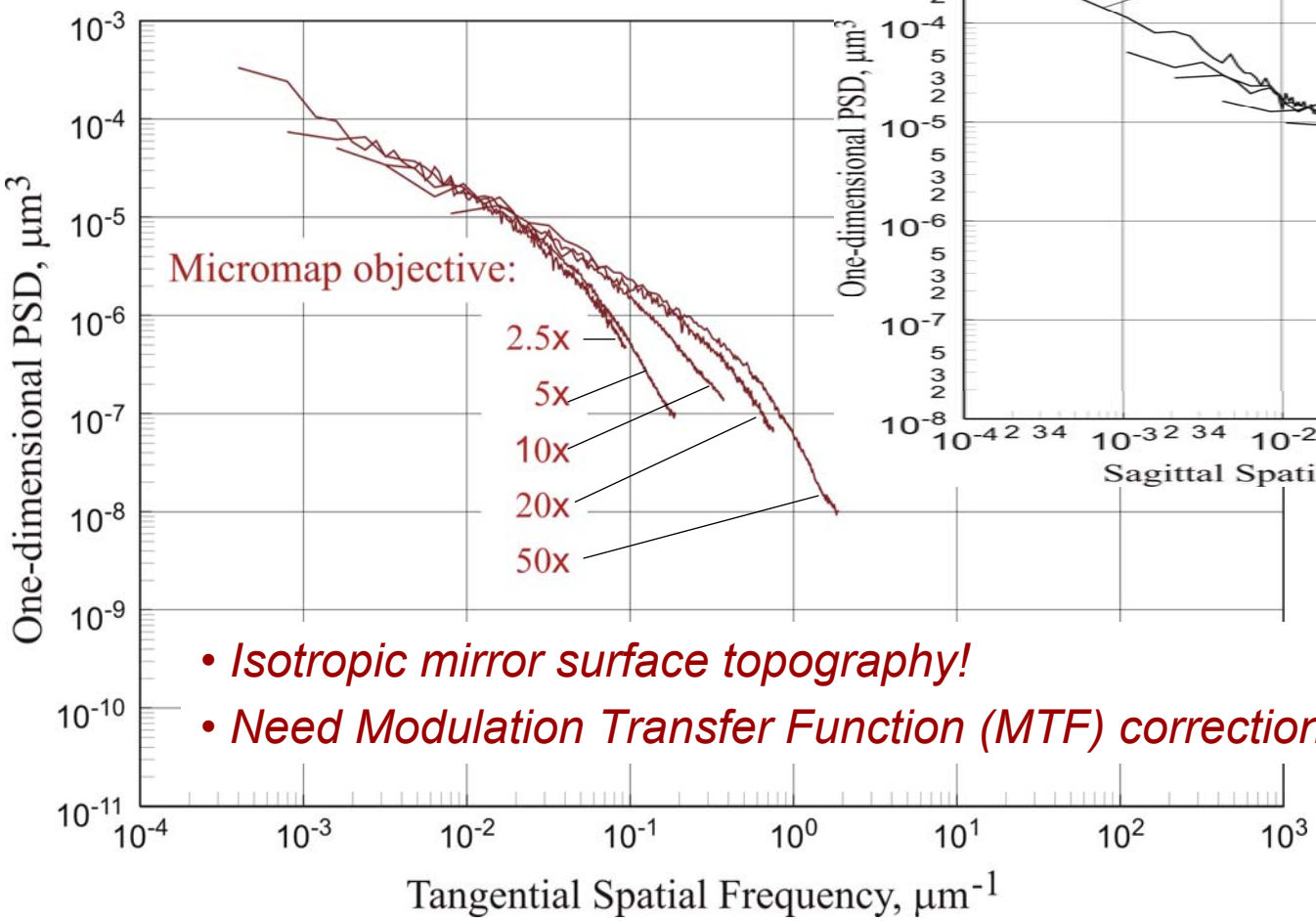
with a Gold coated stainless steel mirror

- ***Micromap-570 Interferometric Microscope***

Sagittal PSD spectra



Tangential PSD spectra



- *Isotropic mirror surface topography!*
- *Need Modulation Transfer Function (MTF) correction*

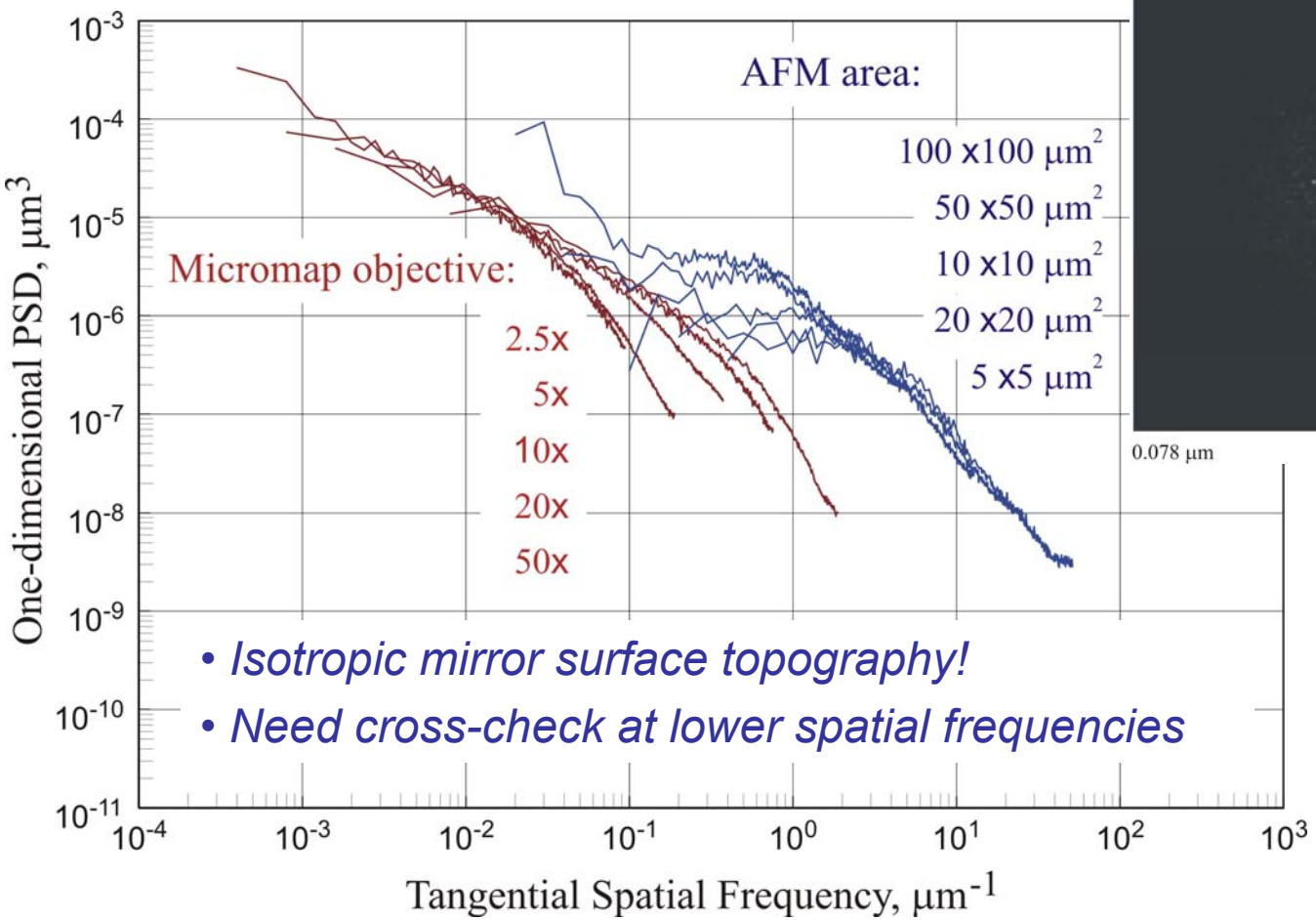
The Micromap CCD camera read-out asymmetry was corrected

SPIE 5858-10

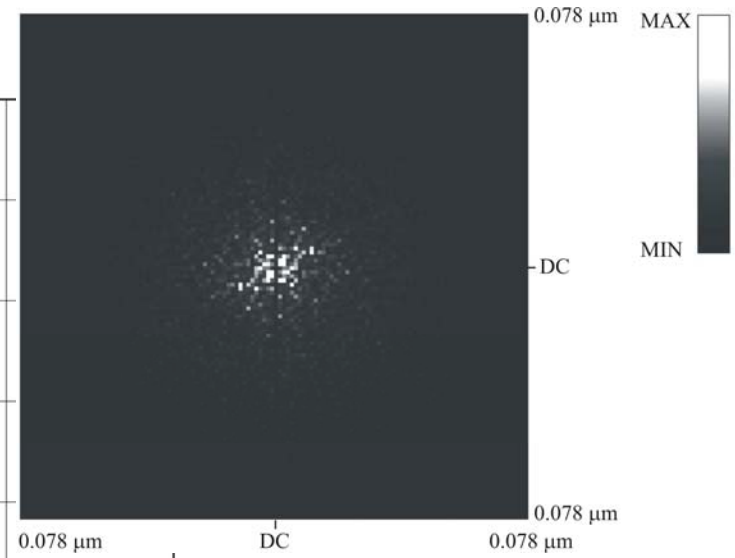
PSD measurements

with a Gold coated stainless steel mirror

- *Micromap-570 Interferometric Microscope*
- ***Digital Instruments Dimension 3100 Atomic Force Microscope***



- *Isotropic mirror surface topography!*
- *Need cross-check at lower spatial frequencies*

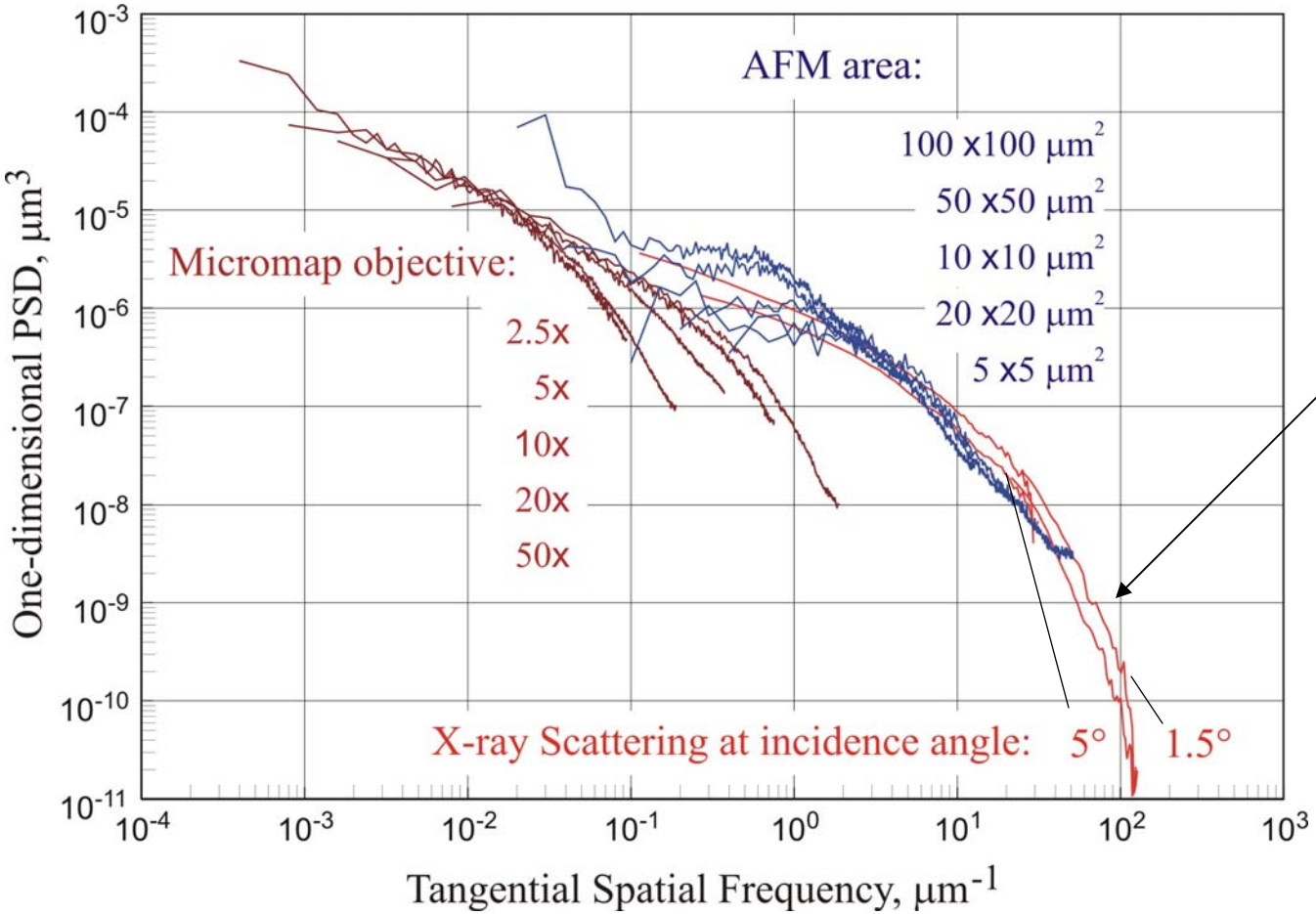


2D PSD distribution measured over 5 × 5 μm² area with 512 × 512 elements (0.2 - 50 μ m⁻¹)

PSD measurements

with a Gold coated stainless steel mirror

- *Micromap-570 Interferometric Microscope*
- *Digital Instruments Dimension 3100 Atomic Force Microscope*
- ***X-ray Reflectivity and Scattering Facility at the LBNL Center for X-ray Optics***

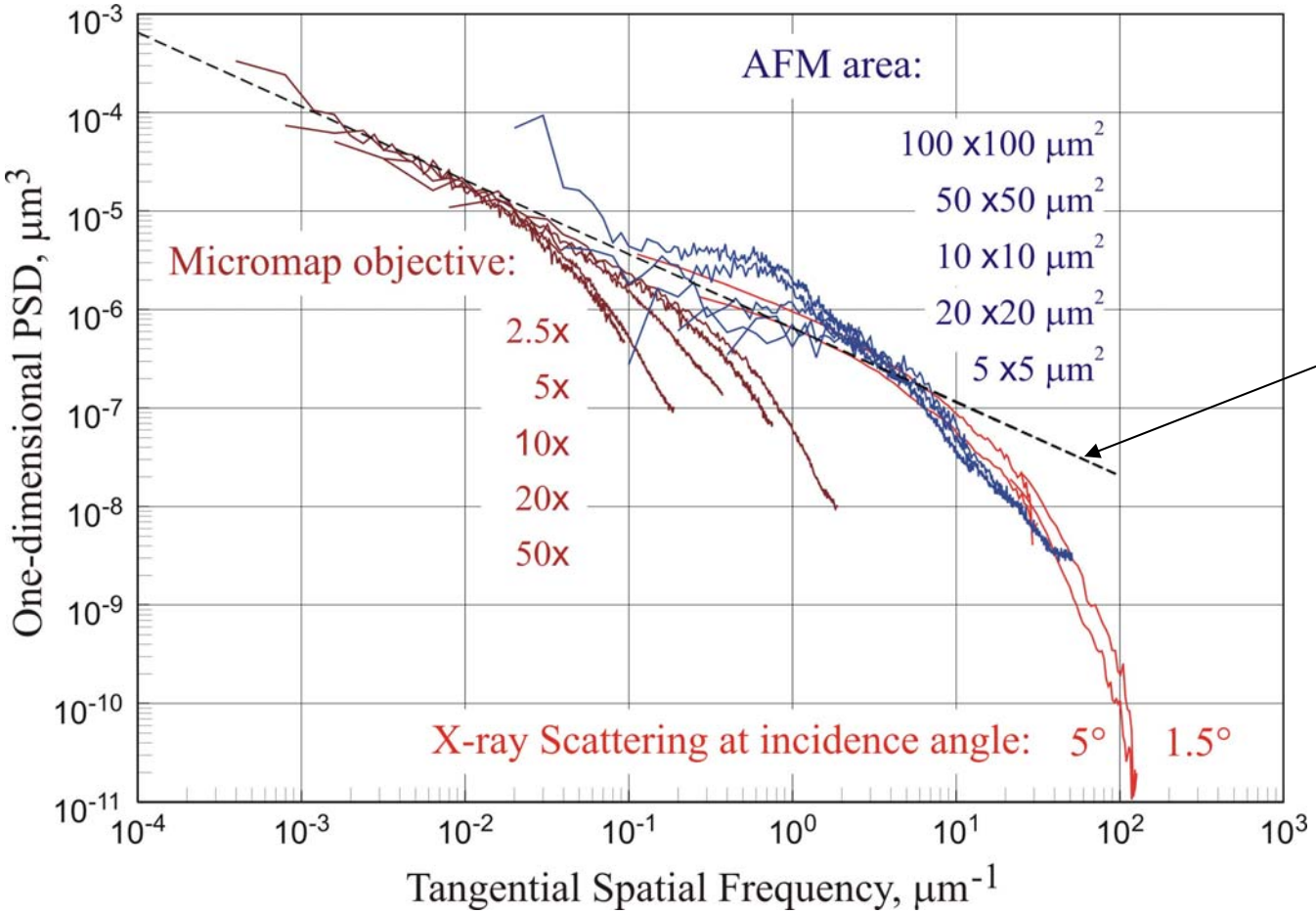


PSD spectra are extracted from the X-ray reflectivity and scattering measurements, assuming isotropic mirror surface topography

Comparison of the PSD measurements

with a Gold coated stainless steel mirror

- ✓ *Essentially consistent results*
  - ✓ *Inverse-power-law approximation at spatial frequencies from 3×10^{-4} to 10 μm^{-1}*
    - ✓ *Possibility to determine 2D MTF of the Micromap and to correct 2D PSD*



$$S_1(f_x) = S_1(1)\,(f_x)^{-\gamma}$$
$$S_1(1) \cong 6.5\cdot10^{-7}$$
$$\gamma \cong 0.75$$

Estimation of the Micromap 2D MTF

from PSDs of a Gold coated stainless steel mirror

$$\left[2D\,MTF(f_x, f_y)\right]^2 = \frac{S_2^M(f_x, f_y)}{S_2^S(f_x, f_y)}$$

1. Surface isotropic topography:

$$S_2(f) = -\frac{1}{2\pi}\int_f^{\infty} \frac{1}{\sqrt{f_x^2 - f^2}} \frac{S_1(f_x)}{df_x} df_x$$

$$f^2 = f_x^{\,2} + f_y^{\,2}$$

2. Inverse-power-law approximation:

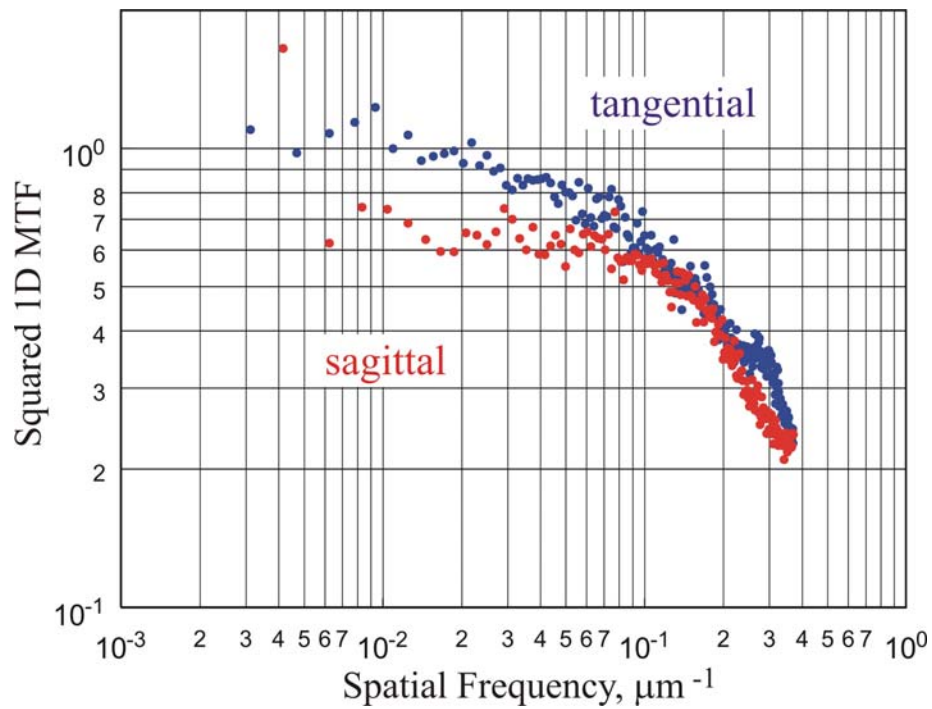
$$S_2^S(f_x, f_y) = \frac{\Gamma[(\gamma+1)/2]\,S_1(1)}{2\sqrt{\pi}\,\Gamma[\gamma/2]\,(f_x^{\,2} + f_y^{\,2})^{(\gamma+1)/2}}$$

$$\left.\begin{aligned} S_1(f_x) &= S_1(1)\,(f_x)^{-\gamma} \\ S_1(1) &\cong 6.5\cdot 10^{-7} \\ \gamma &\cong 0.75 \end{aligned}\right\} \quad \begin{aligned} \Gamma[(\gamma+1)/2] &\cong 2.37 \\ \Gamma[\gamma/2] &\cong 1.09 \end{aligned}$$

[J.C. Stover, *Optical Scattering*, SPIE Press, 1995]

Problem:

The found 2D MTF contains the noise of the measured PSD



Solution:

To smooth the 2D MTF

Estimation of the Micromap 2D MTF

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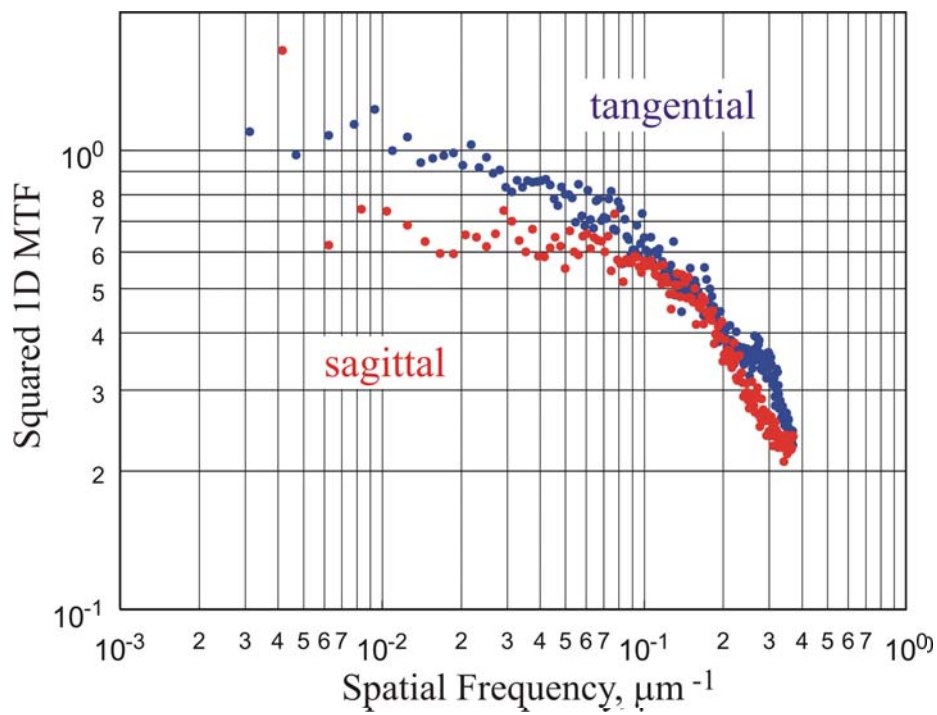
To smooth the 2D MTF

3. Assume isotropic topography of the MTF and fit the 1D MTFs



Estimation of the Micromap 2D MTF

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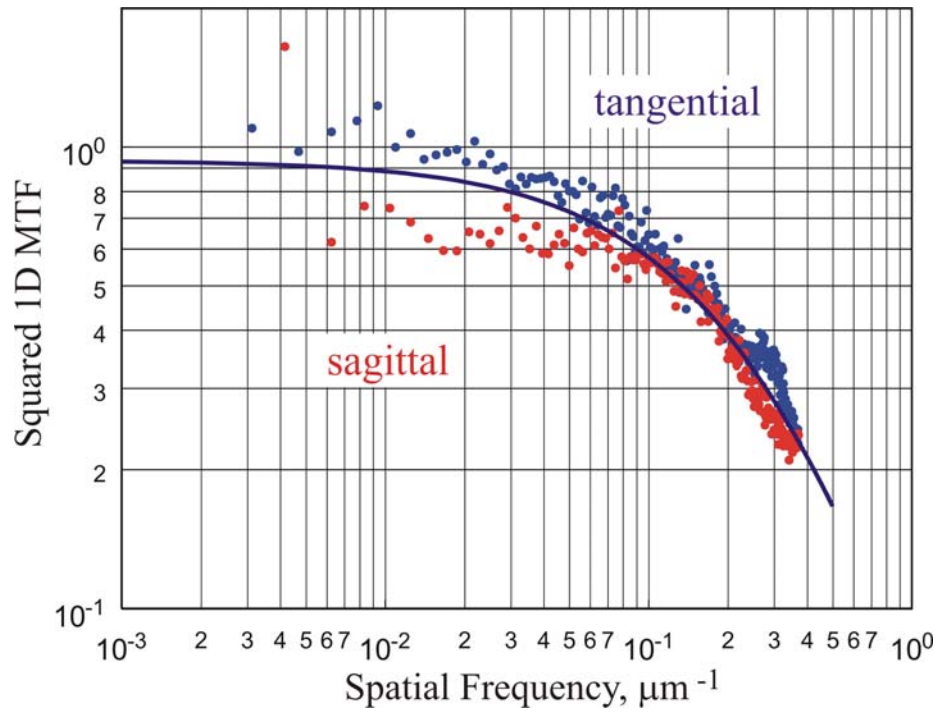

with an analytical function:

$$\left[1D\,MTF(f_x)\right]^2 \propto \frac{1}{(1+a\,f_x)^2}$$

$$a \cong 2.73\mu\text{m}$$

Estimation of the Micromap 2D MTF

from PSDs of a Gold coated stainless steel mirror

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[J.C. Stover, *Optical Scattering*, SPIE Press, 1995]

3. Assume isotropic topography of the MTF and fit the 1D MTFs with an analytical function:

$$\left[1D\,MTF(f_x)\right]^2 \propto \frac{1}{(1+a\,f_x)^2}$$

$$a \cong 2.73\ \mu\text{m}$$

4. Integrate 1D MTF to get smooth 2D MTFs

$$\left[2D\,MTF(f)\right]^2 = -\frac{1}{2\pi}\int_f^\infty \frac{1}{\sqrt{f_x^2 - f^2}}\frac{\left[1D\,MTF(f_x)\right]^2}{df_x}df_x$$

$$f^2 = f_x^{\,2} + f_y^{\,2}$$

5. Apply the 2D MTF to correct the 2D PSDs

$$S_2^S(f_x,f_y) = \frac{S_2^M(f_x,f_y)}{MTF^2(f_x,f_y)}$$

MTF Correction of the Micromap PSDs

measured with the *Gold coated stainless steel mirror*

$$[2D\,MTF(f_x, f_y)]^2$$

1. Surface isotrop

$$S_2(f) = -\frac{1}{2\pi}\int_f^{\infty} \frac{}{\sqrt{f}}$$

2. Inverse-power-

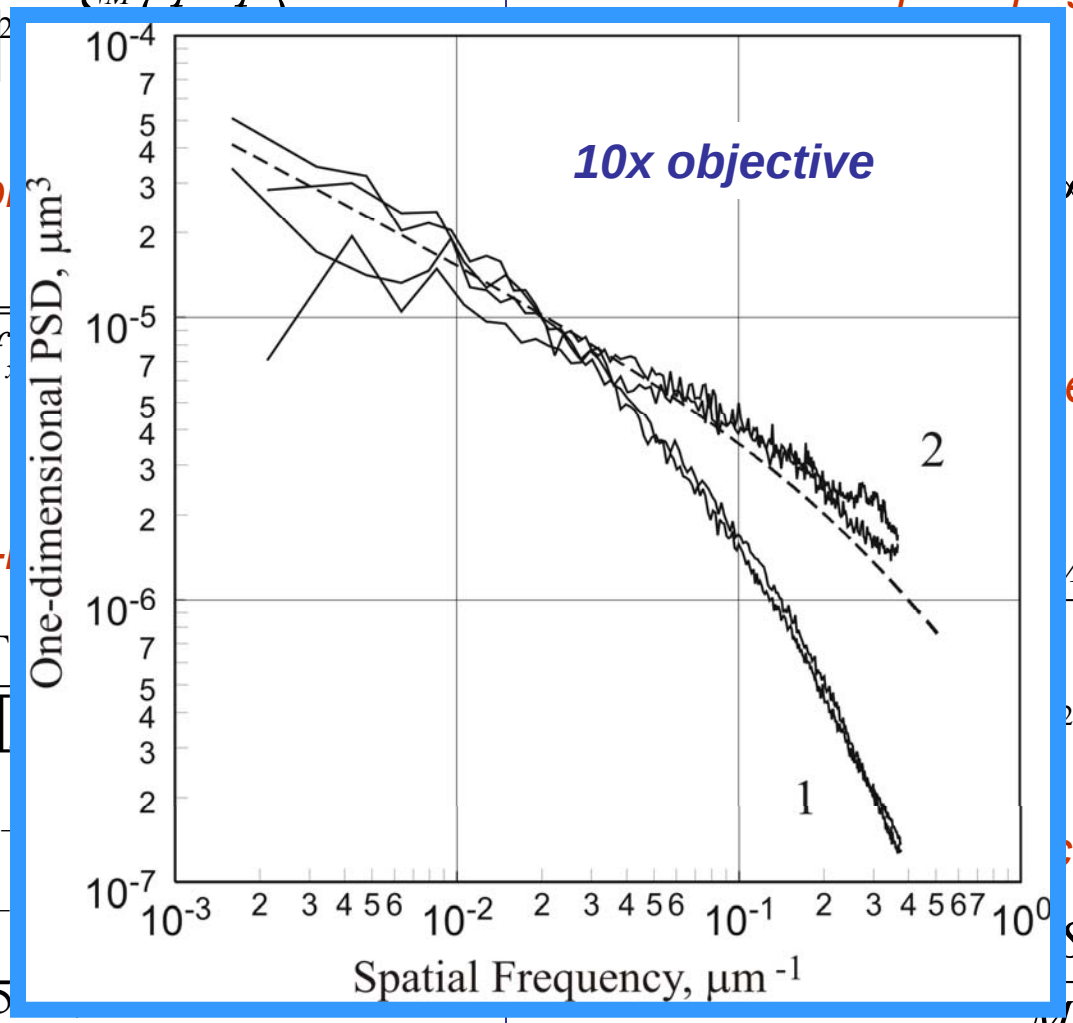
$$S_2^S(f_x, f_y) = \frac{\Gamma}{2\sqrt{\pi}\,\Gamma[}$$

$$S_1(f_x) = S_1(1)\,(f_x)$$

$$S_1(1) \cong 6.5 \cdot 10$$

$$\gamma \cong 0.75$$

[J.C. Stover, *Optical Scattering*, SPIE Press, 1995]



3. Assume isotropic topography of the MTF
an analytical

$$\propto \frac{1}{(1 + a\,f_x)^2}$$

et smooth 2D MTFs

$$\frac{MTF(f_x)]^2}{df_x}\,df_x$$

$$^2 = f_x^{\,2} + f_y^{\,2}$$

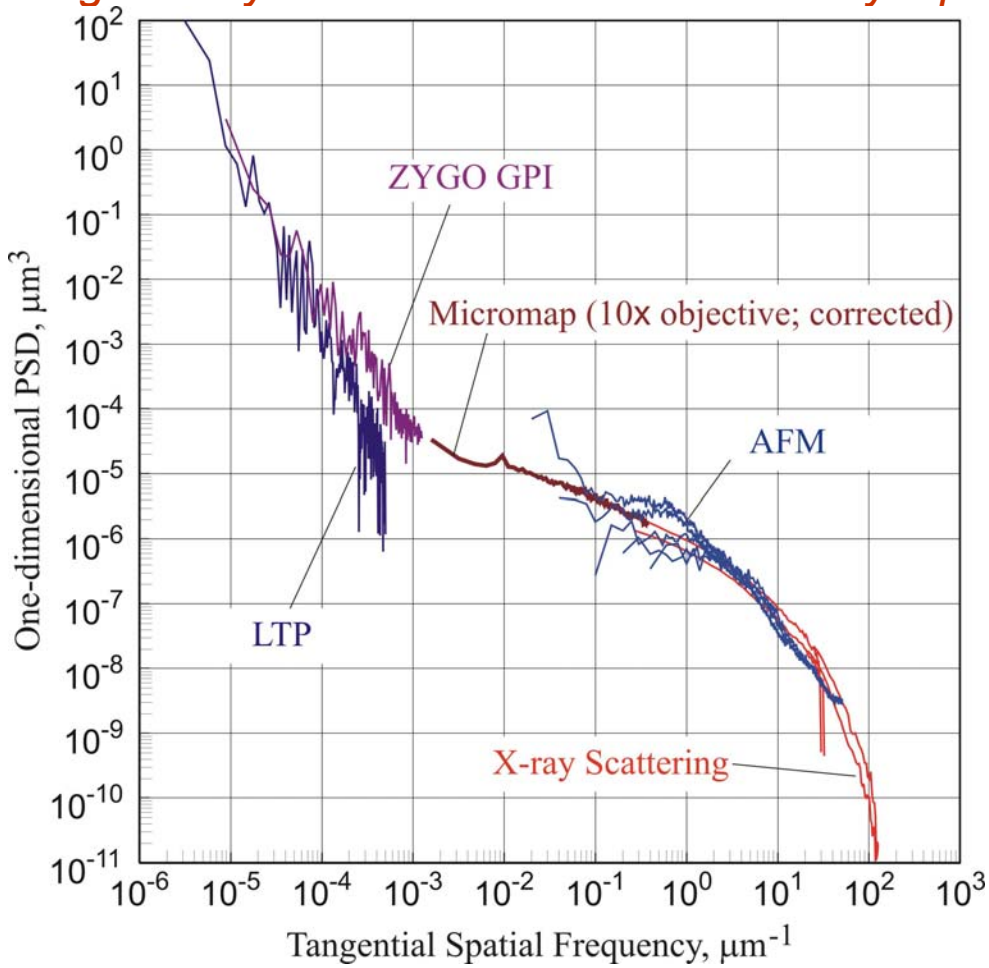
correct the 2D PSDs

$$\frac{S_2^M(f_x, f_y)}{MTF^2(f_x, f_y)}$$

PSD measurements SUMMARY

with a Gold coated stainless steel mirror

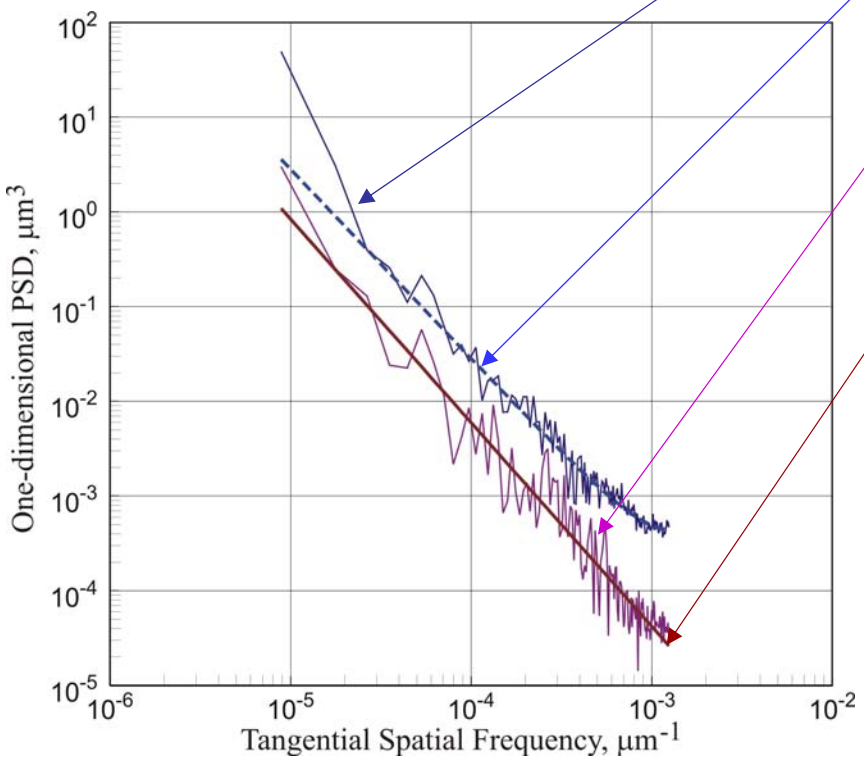
- *Micromap-570 Interferometric Microscope*
- *Digital Instruments Dimension 3100 Atomic Force Microscope*
- *X-ray Reflectivity and Scattering Facility at the LBNL Center for X-ray Optics*
- *ZYGO GPI interferometer*
- *Long Trace Profiler LTP-II*



PSD measurements

with a Gold coated stainless steel mirror

- ***ZYGO GPI interferometer***



Data detrended with piston and tilt

Calculated PSD of a tilted ideal flat surface

Same data detrended with a second power polynomial function

Fitted inverse-power-law approximation with power index of -2.15

Problem:

Lower frequency PSD spectra, with high log-log steepness approximated with an inverse-power-law with large ($\gamma \geq 2$) index, strongly depend on detrending.

Conclusions

✓ *Consistence and mutual supplementation of PSD measurements performed with different instruments was demonstrated.*

✓ *A cross-check measurement was used for finding the MTF of the Micromap.*

✓ *The MTF correction procedure takes advantage of the isotropic topography of the test surface, the isotropy of the 2D MTF, and the inverse-power-law approximation of the PSD distribution observed for the spatial frequency range, corresponding to the Micromap.*

✓ *Lower frequency PSD measurements with the LTP and ZYGO GPI require additional cross-checks with a higher resolution instrument to resolve discrepancies and validate the results*

✓ ***Next:*** *- Application of the PSD data to X-ray scattering calculations*

- Investigation of the reliability of lower frequency PSD measurements

- Development of a test surface standard for 2D PSD measurement

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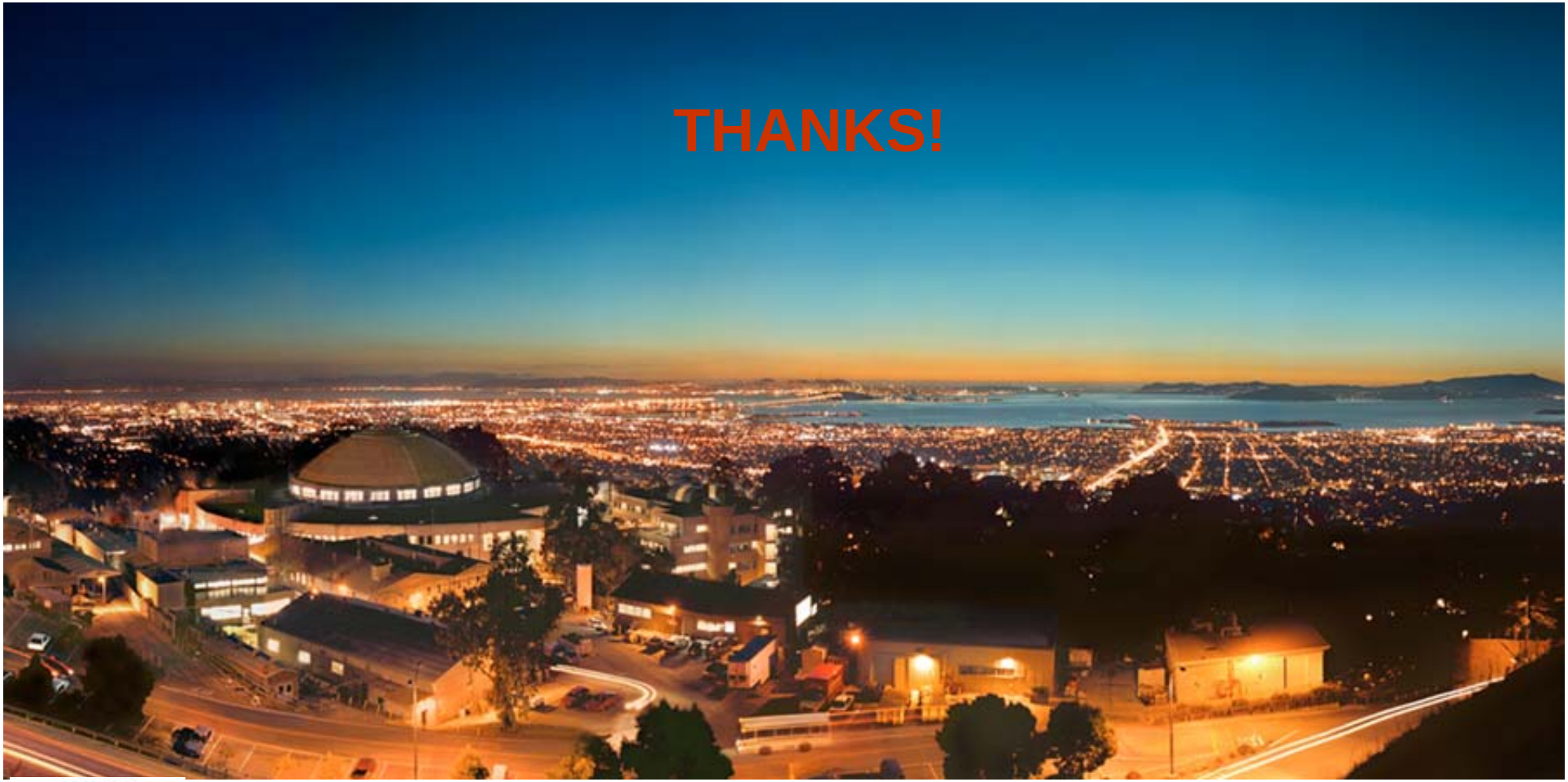
THANKS!