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Positioning Errors of Pencil-beam Interferometers for Long Trace Profilers

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Outline

- ***Introduction***

  LTP based on pencil-beam interferometer

- ***Motivation***

  Practical and
  Philosophical reasons

- ***Method***

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- ***Application of regression analysis to LTP fitting***

  Estimation of positioning error due to
  - *photo-detector dark signal*
  - *photo-detector read-out error*
  - *resolution of ADC of the detector*
  - *photo-response non-uniformity*
  - *non-uniformity of the pixel pitch*
  - *signal shot noise*

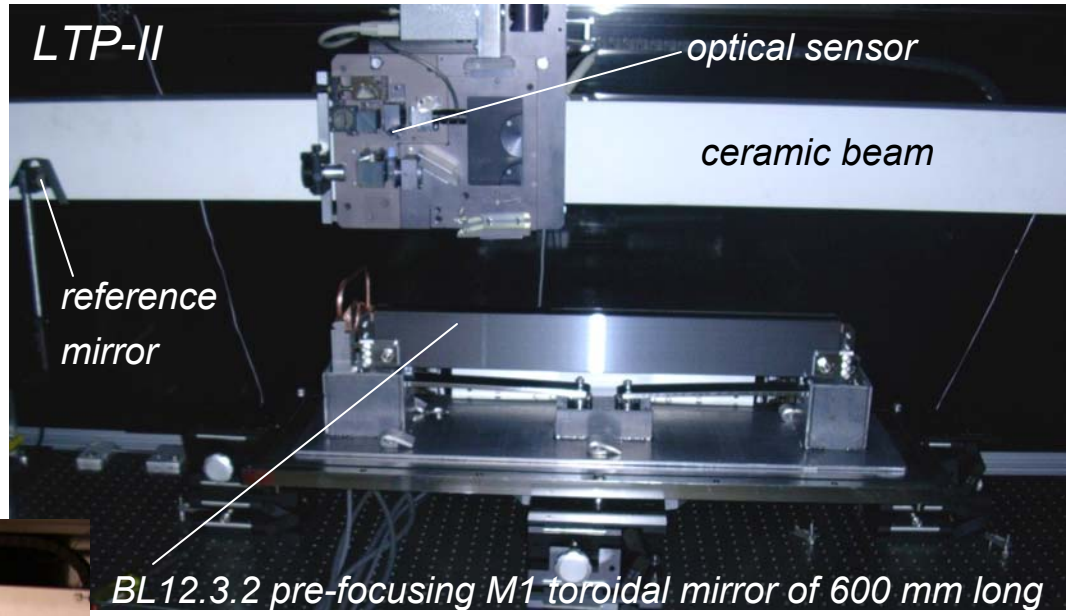
- ***Summary and Conclusions***

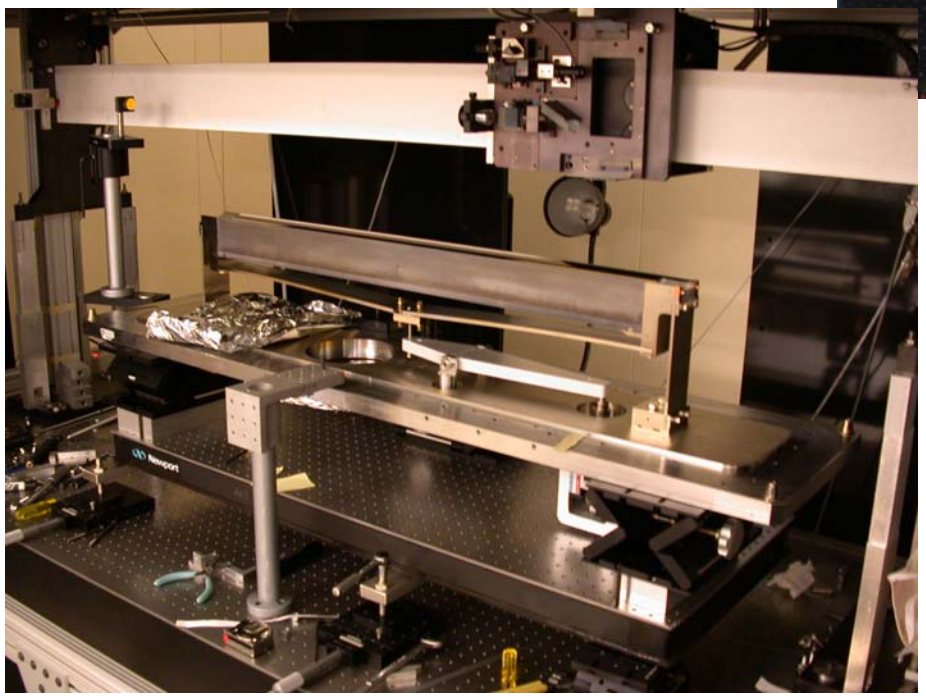
Long trace profiler LTP-II

Surface characterization at lower spatial frequencies

Mainframe Performance:

- *1D slope-trace measuring instrument*
- *For mirrors as long as 1 m*
- *Accuracy: ~1 µrad*
- *Spatial Sampling: 1 mm*
- *Digitization: 16 bits*
- *Indispensable for bendable mirror adjustment, characterization, and calibration*





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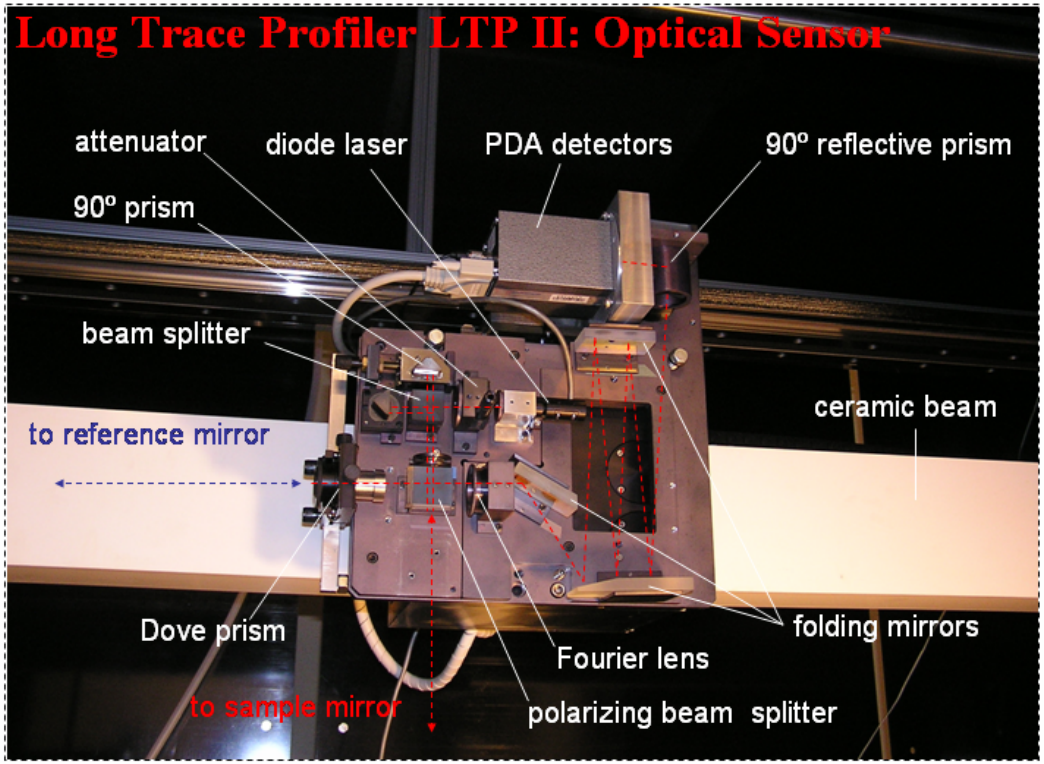
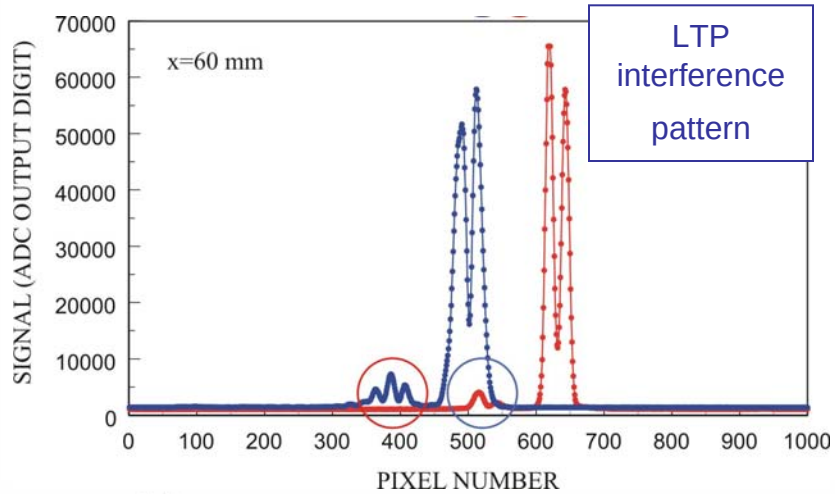
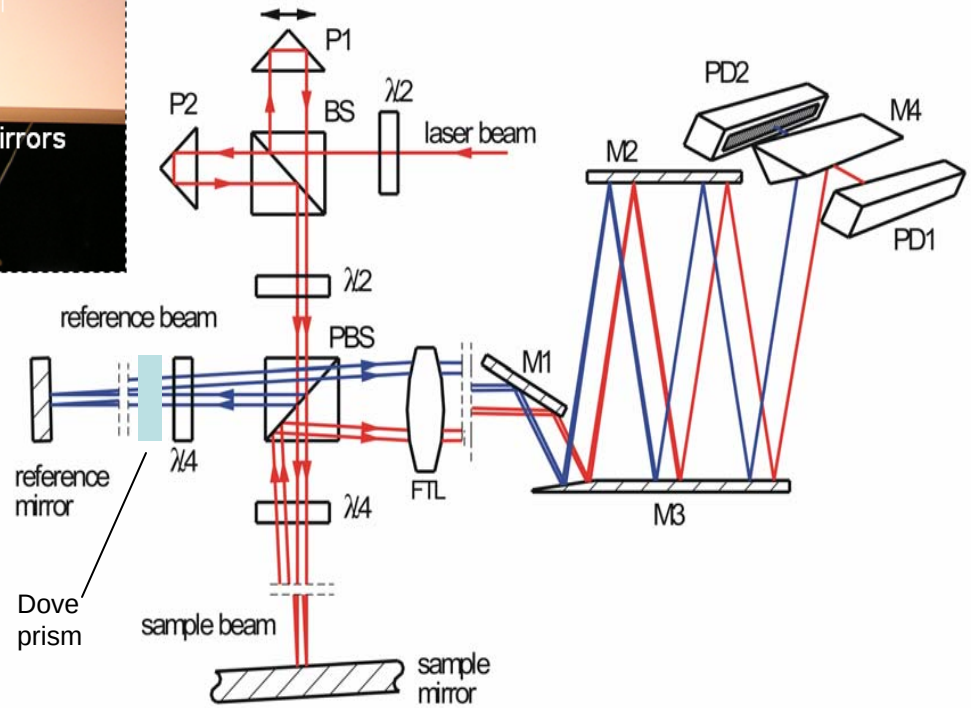
Foundations of Long trace profiler LTP-II:

Pencil-beam interferometer





LTP interference pattern



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Motivation

Practical reasons:

to provide reliable error-bars for measured slope traces

to understand the error budget of LTP measurement, and

to determine a course of LTP improvement

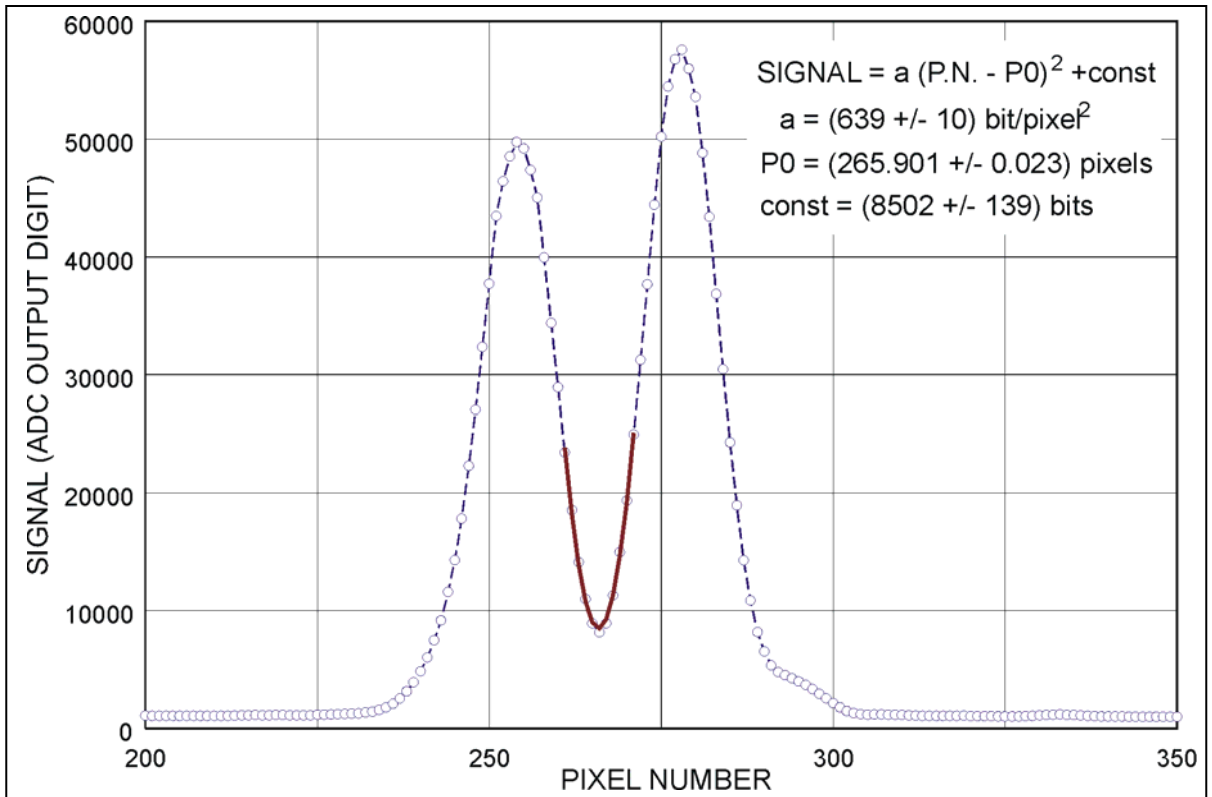
'Philosophical reason:'

"…Fathom the unfathomable…"

F. Nietzsche, *Thus Spoke Zarathustra* (1885)

put error bars in LTP data

Problem to be solved



- *Detector is a linear photo-diode array of 1024 pixels with a pitch of 25 μm and sagittal width of 2.5 mm.*
- *The focal distance of the LTP II is F=1.250 m; a coefficient of detector-position-to-slope-angle transformation is 1/(2F), where F is given in meters, and slope angle and beam position on the detector are measured in μrad and μm, respectively.*

Get general relations to be used for optimization of the LTP design, measurement process and fitting procedure

Positioning includes:

1. Find best fit parameters

$$\theta_1^*, \quad \theta_2^*, \quad \theta_3^*$$

of the fitting function:

$$\eta(x,\theta) = \theta_0 + \theta_1 x + \theta_2 x^2$$

2. Estimate the errors (dispersions) of the parameters:

$$s^2(\theta_1^*), \quad s^2(\theta_2^*), \quad s^2(\theta_3^*)$$

3. Find the position of the central minimum:

$$x_{\min} = -\frac{\theta_1^*}{2\theta_2^*}$$

4. Estimate the error (dispersion) of the central minimum position:

$$s^2(x_{\min}) = x_{\min}^2 \left\{ \frac{s^2(\theta_1^*)}{(\theta_1^*)^2} + \frac{s^2(\theta_2^*)}{(\theta_2^*)^2} \right\}$$

Method

Review of the linear regression method

Conditional distribution of two variables: a mean value of y vs (x,θ)

$$\mathrm{E}(y \mid x) = \eta(x,\theta)$$

Observation: $y_i = \eta(x_i,\theta) + \varepsilon_i$; *where:* ε_i *is the error variable*

$$\eta(x,\theta) = \theta_0 f_0(x) + \theta_1 f_1(x) + \ldots + \theta_r f_r(x)$$

is a linear combination of functions.

Simple transformations to apply Method of Least Squares:

$$\varepsilon_i = y_i - \theta_0 f_0(x_i) - \theta_1 f_1(x_i) - \ldots - \theta_r f_r(x_i),\ i = 1,2,\ldots,n$$

$$S = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} (y_i - \theta_0 f_0(x_i) - \theta_1 f_1(x_i) - \ldots - \theta_r f_r(x_i))^2$$

$$\frac{\partial S}{\partial \theta_\rho} = 0,\quad \rho = 0,1,\ldots,r.$$

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$$\frac{\partial S}{\partial \theta_\rho} = 0,\ \ \rho = 0,1,\ldots,r.$$

Introduce a regression matrix:

$$\hat{A} = \begin{bmatrix} f_0(x_1) & f_1(x_1) & f_2(x_1)\ldots & f_r(x_1) \\ f_0(x_2) & f_1(x_2) & f_2(x_2)\ldots & f_r(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ f_0(x_n) & f_1(x_n) & f_2(x_n)\ldots & f_r(x_n) \end{bmatrix}$$

Then:

$$\hat{\varepsilon} = \hat{y} - \hat{A}\hat{\theta}$$

$$S = \hat{\varepsilon}'\hat{\varepsilon} = \hat{y}'\hat{y} - 2\hat{\theta}'\hat{A}'\hat{y} + \hat{\theta}'\hat{A}'\hat{A}\hat{\theta}$$

$$-\hat{A}'\hat{y} + \hat{A}'\hat{A}\hat{\theta} = 0;$$

Solution:

$$\hat{\theta}^* = (\hat{A}'\hat{A})^{-1}\hat{A}'\hat{y}$$

$$D\hat{\theta}^* = ((\hat{A}'\hat{A})^{-1}\hat{A}')D\hat{y}((\hat{A}'\hat{A})^{-1}\hat{A}')' = \sigma^2(\hat{A}'\hat{A})^{-1}$$

$$D\hat{y} = \sigma^2\hat{I}$$

Application of regression analysis to LTP fitting

Observation: $y_i = \eta(x_i, \theta) + \varepsilon_i$; *where:* ε_i *is the error variable*

$$\eta(x,\theta) = \theta_0 f_0(x) + \theta_1 f_1(x) + \ldots + \theta_r f_r(x)$$

is a linear combination of functions.

For LTP fitting:

$$f_\rho(x) = x^\rho : \quad \eta(x,\theta) = \theta_0 + \theta_1 x + \theta_2 x^2$$

Choosing:

$$(x_i) = (-m, -(m-1), \cdots, -i, \cdots, -1, 0, 1, \cdots, i, \cdots, (m-1), m)\, pixel$$

Get in the case of intensity independent noise:

$$s^2(\theta_0^*) = \sigma^2 \frac{9m^2 + 9m - 3}{8m^3 + 12m^2 - 2m - 3} = \sigma^2 s_0^2(m),$$

$$s^2(\theta_1^*) = \sigma^2 \frac{3}{2m^3 + 3m^2 + m} = \sigma^2 s_1^2(m),$$

$$s^2(\theta_2^*) = \sigma^2 \frac{45}{m(2m+1)(4m^3 + 8m^2 + m - 3)} = \sigma^2 s_2^2(m).$$

Regression matrix:

$$\hat{A} = \begin{bmatrix} f_0(x_1) & f_1(x_1) & f_2(x_1) \ldots & f_r(x_1) \\ f_0(x_2) & f_1(x_2) & f_2(x_2) \ldots & f_r(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ f_0(x_n) & f_1(x_n) & f_2(x_n) \ldots & f_r(x_n) \end{bmatrix}$$

$$\hat{A} = \begin{bmatrix} 1 & x_1 & x_1^2 \\ 1 & x_2 & x_2^2 \\ \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 \end{bmatrix}$$

Solution:

$$\hat{\theta}^* = (\hat{A}'\hat{A})^{-1}\hat{A}'\hat{y}$$

$$D\hat{\theta}^* = ((\hat{A}'\hat{A})^{-1}\hat{A}')\, D\hat{y}\, ((\hat{A}'\hat{A})^{-1}\hat{A}')'$$

$$= \sigma^2 (\hat{A}'\hat{A})^{-1}$$

$$\boxed{D\hat{y} = \sigma^2 \hat{I}}$$

Positioning errors due to intensity independent noise:

photo-detector dark signal and read-out error

$$s^2(\theta_0^*) = \sigma^2 \frac{9m^2 + 9m - 3}{8m^3 + 12m^2 - 2m - 3} = \sigma^2 s_0^2(m),$$

$$s^2(\theta_1^*) = \sigma^2 \frac{3}{2m^3 + 3m^2 + m} = \sigma^2 s_1^2(m),$$

$$s^2(\theta_2^*) = \sigma^2 \frac{45}{m(2m+1)(4m^3 + 8m^2 + m - 3)} = \sigma^2 s_2^2(m).$$

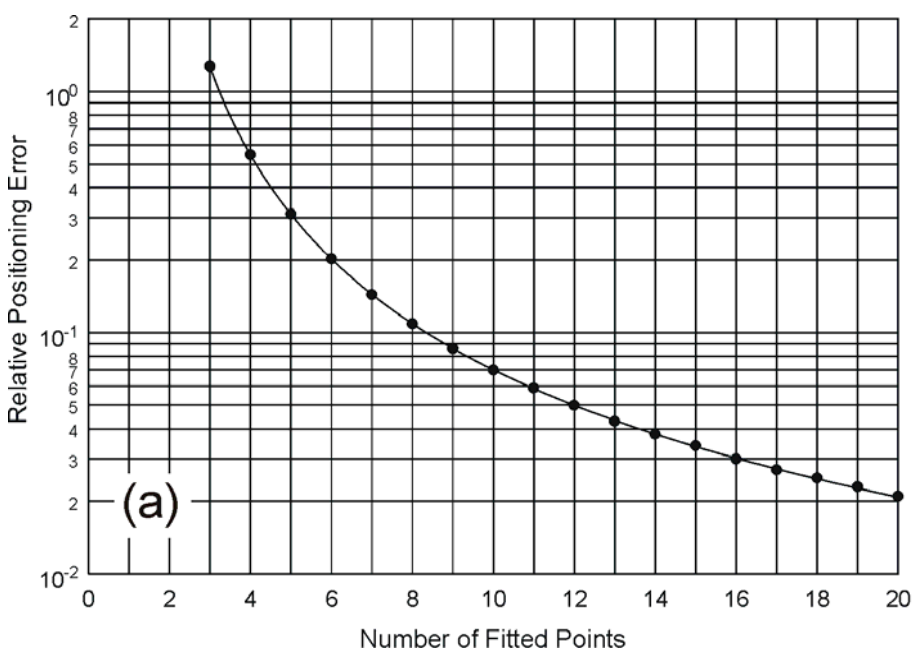
$$(x_i) = (-m, -(m-1), \cdots, -i, \cdots, -1, 0, 1, \cdots, i, \cdots, (m-1), m)\, pixel$$

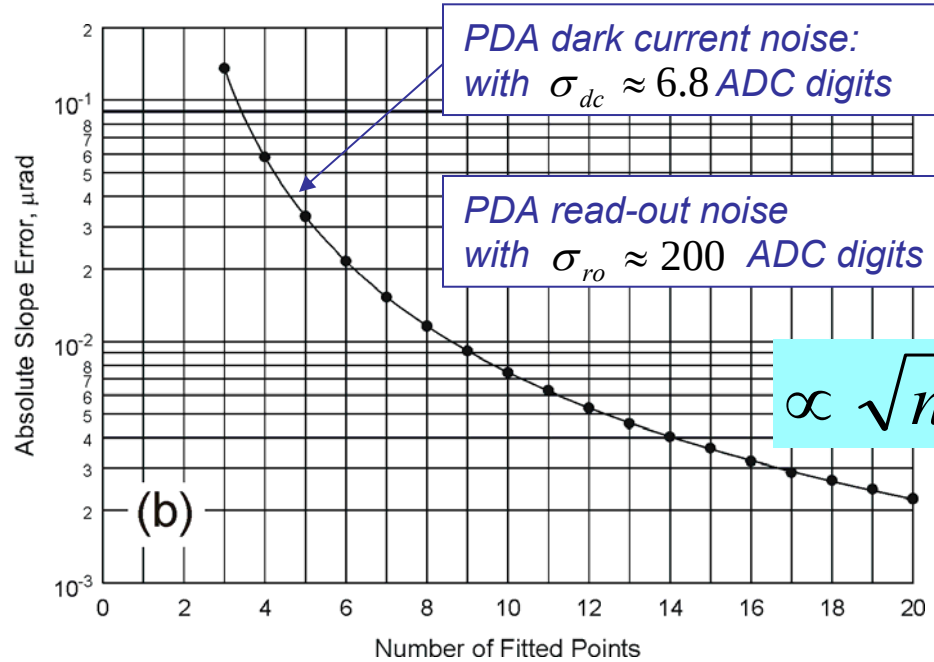
position of the central minimum:

$$x_{\min} = -\frac{\theta_1^*}{2\theta_2^*}$$

dispersion of $x_{\min}$:

$$s^2(x_{\min}) = x_{\min}^2 \left\{ \frac{s^2(\theta_1^*)}{(\theta_1^*)^2} + \frac{s^2(\theta_2^*)}{(\theta_2^*)^2} \right\}$$





PDA dark current noise: with $\sigma_{dc} \approx 6.8$ ADC digits

PDA read-out noise with $\sigma_{ro} \approx 200$ ADC digits

$\propto \sqrt{n}$

Positioning error due to

photoresponse non-uniformity of the photo-diode array

non-uniformity of a PDA (CCD) photoresponse for the neighbor pixels is usually specified to be about 2-3%

$$y_i = (1+\xi_i)\left(\theta_0 + \theta_1 x_i + \theta_2 x_i^2\right) = \theta_0 + \theta_1 x_i + \theta_2 x_i^2 + \varepsilon_i,$$

$$\varepsilon_i = \xi_i \left(\theta_0 + \theta_1 x_i + \theta_2 x_i^2\right) \approx \xi_i \left(\theta_0^* + \theta_1^* x_i + \theta_2^* x_i^2\right) \equiv \xi_i \; y_i^*,$$

Intensity dependent error:

$$D(\varepsilon_i) \approx D(\xi_i)\left(y_i^*\right)^2 = \sigma_{prn}^2 \left(y_i^*\right)^2$$

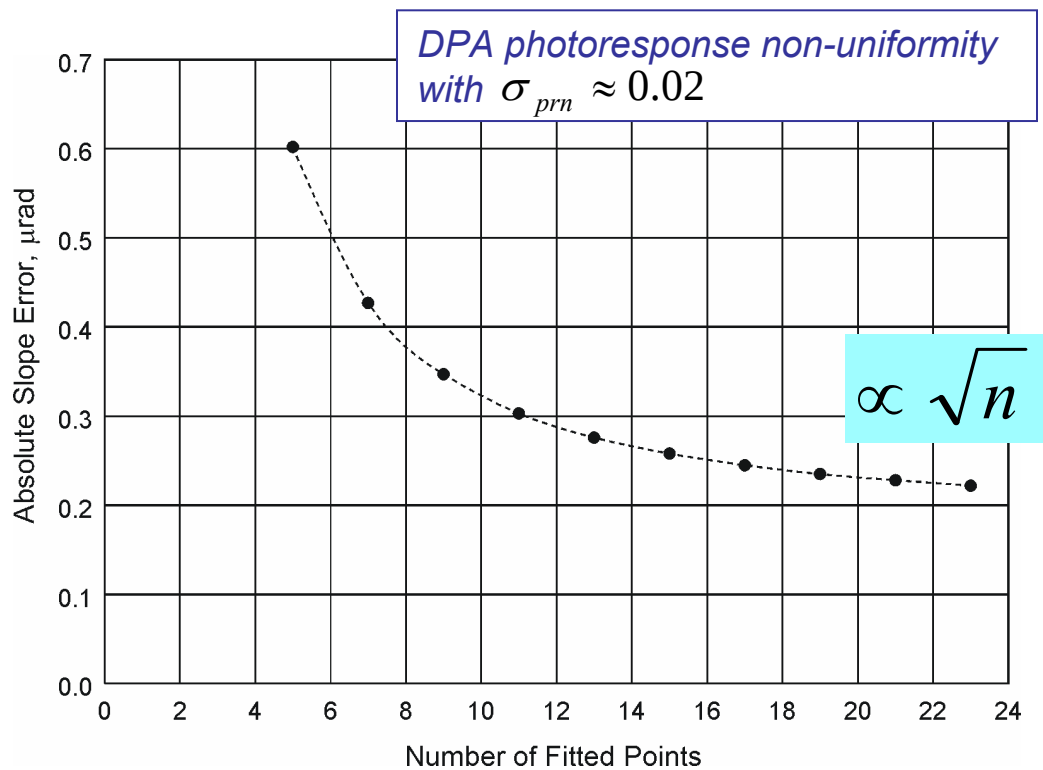
Then, construct the weight matrix $\hat{W}_{prn}$

with elements $\omega_{i,i} = (y_i^*)^2$

Solution:

$$\hat{\theta}^* = (\hat{A}'\hat{W}^{-1}\hat{A})^{-1}\hat{A}'\hat{W}^{-1}\hat{y}$$

$$D(\hat{\theta}) = \sigma^2 (\hat{A}'\hat{W}^{-1}\hat{A})^{-1}$$



NOTE: ***Similar approach can be used to account for*** ***- positioning error due to signal shot noise***

- non-uniformity of the effective pixel pitch

Summary

Table 1. LTP II positioning error due to the different sources of noise.

Noise source	Position error (pixel) at $n = 11$	Slope error (μrad)		Asymptotic with n
		$n = 11$	$n = 23$	
PDA dark signal	$6 \cdot 10^{-4}$	0.006	0.002	$\propto n^{-1/2}$
Read-out noise	$2 \cdot 10^{-2}$	0.2	0.07	$\propto n^{-1/2}$
ADC resolution (at 2^{10} bits)	$2 \cdot 10^{-3}$	0.06	0.02	$\propto n^{-1/2}$
PDA photo-response (at 2%)	$3 \cdot 10^{-2}$	0.3	0.2	$\propto n^{-1/2}$
Pixel pitch non-uniformity (at 10%)	$3 \cdot 10^{-2}$	0.3	0.2	$\propto n^{-1/2}$
Signal shot noise	$2 \cdot 10^{-4}$	0.002	0.001	$\propto n^{-3/4}$

Note:

- ***main contribution to the positioning error comes from the PDA photoresponse and pixel pitch non-uniformity;***
- ***strong dependence on the number of points used for fitting;***
- ***the same error sources should be accounted for in both the sample and the reference arms.***

Conclusions

✓ *General model for LTP positioning errors has been developed.*

✓ *Estimation of positioning error due to photo-detector dark signal, photo-detector read-out error, photo-response non-uniformity, non-uniformity of the pixel pitch, signal shot noise has been performed.*

✓ *Recommendations on improvement of the current LTP design and method are suggested:*

- *The LTP Photo-detector has to be precisely calibrated to suppress the error due to photoresponse and pixel pitch non-uniformity;*
- *A PDA (CCD) camera with 2^{10}-bit ADC would be adequate for the LTP measurement with accuracy of ~0.1 μrad;*
- *A PDA with pixel pitch smaller by a factor of k would give $\sqrt{k}$ improvement of positioning accuracy*;
- *A more sophisticated fitting procedure, which allows extending the number of points used for fitting, would significantly improve the positioning accuracy*.

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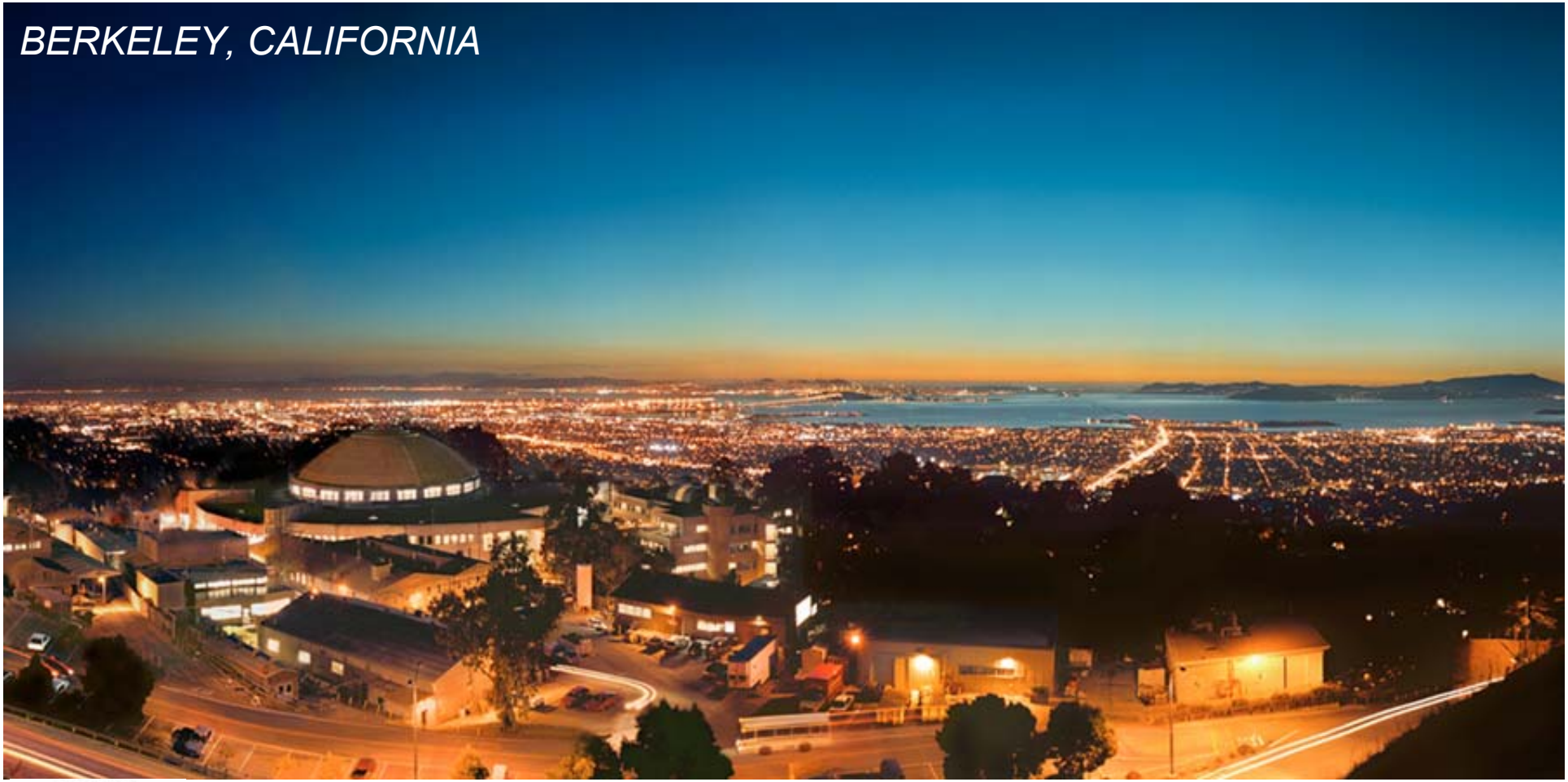
THANKS!



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