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New Procedures for the Adjustment of Elliptically Bent Mirrors with the Long Trace Profiler

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Outline

- **Motivation**:

  Micro-focusing often accomplished with KB pairs
  KB Pairs most economically implemented with bendable mirrors
  Systematic method of adjustment needs described

- **Use LTP to adjust bendable mirrors**

  Review theory of bendable mirrors—demonstrate linearity
  Show complete method based on regression analysis
  Show 4 radii method
  Derive wave front ray trace as a check

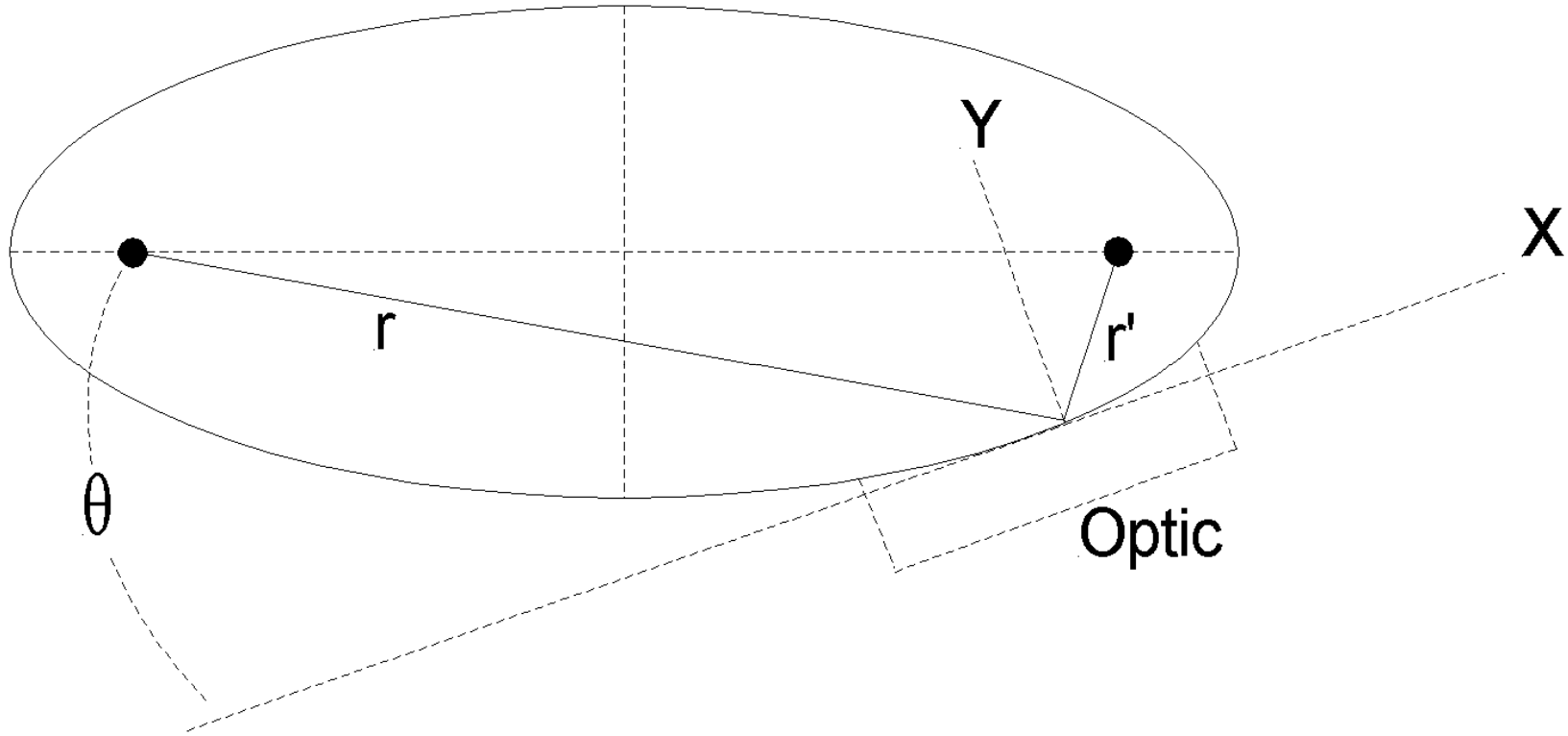
- **Give detailed example of adjustment of a bendable mirror**

Theory of Bendable Mirrors

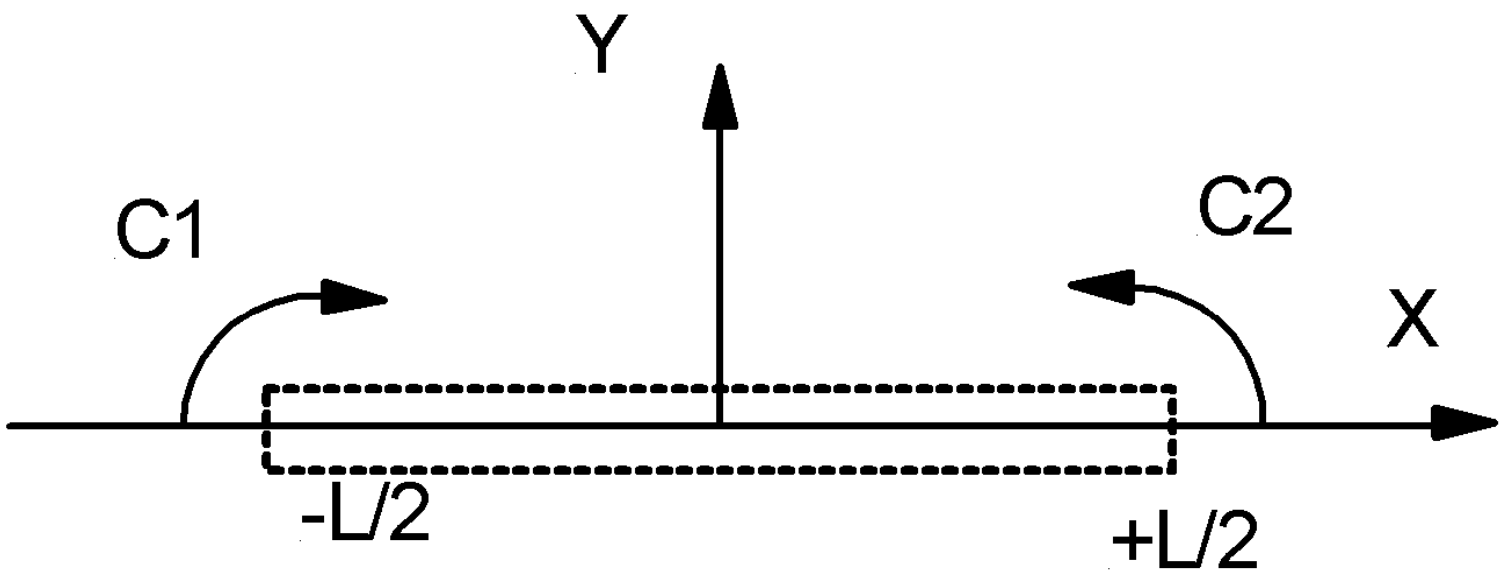
$$E\,I(x)\frac{d^2y}{dx^2} = M(x)$$

- *X* is the coordinate along the beam, the tangential direction.
- *Y* is in the direction of the deflection of the beam, the surface normal.
- *E* is Young's modulus of the mirror material.
- *I(x)* is the moment of inertia as a function of position along the beam, or mirror.
- *M(x)* is the bending moment.

Geometry of Tangential Elliptical Mirror



Geometry of Mirror Bending



General Theory I

Add effects of couples

$$EI(x)\frac{d^2y}{dx^2} = \frac{C_1 + C_2}{2} - \frac{C_1 - C_2}{L}x\,.$$

Make definitions

$$g_1(x) = \left(\frac{1}{2} - \frac{1}{L}x\right)\frac{1}{EI(x)} \qquad g_2(x) = \left(\frac{1}{2} + \frac{1}{L}x\right)\frac{1}{EI(x)}$$

Add definitions

$$\frac{d^2y}{dx^2} = C_1 g_1(x) + C_2 g_2(x)$$

General Theory II

Integrate to slope

$$\alpha(x,\mathbf{C}) \equiv \frac{dy}{dx} = C_0 + C_1 f_1(x) + C_2 f_2(x)$$

Using new definitions

$$f_1(x) = \int g_1(x)\,dx \quad and \quad f_2(x) = \int g_2(x)\,dx;$$

The slope is a linear function of f1 and f2

Ellipse Theory I

Let superscript 0 indicate ellipse

$$\alpha^0(x,C) = C_0^0 + C_1^0 f_1(x) + C_2^0 f_2(x)$$

Let ellipse be imperfect

$$\Delta\alpha(x,C) = \Delta C_0 + \Delta C_1 f_1(x) + \Delta C_2 f_2(x)$$

Measure the optic with the LTP

$$\delta\alpha_{20}(x_i) = \Delta C_0 + \Delta C_1\, f_1(x_i) + \Delta C_2\, f_2(x_i) + \varepsilon_{0i}$$

Move one bender adjustment, C1, and re-measure

$$\delta\alpha_{21}(x_i) = \Delta C_0 + (\Delta C_1 + \delta C_1)\, f_1(x_i) + \Delta C_2\, f_2(x_i) + \varepsilon_{1i}$$

Subtract

$$\delta\alpha_{21}(x_i) - \delta\alpha_{20}(x_i) = \delta C_1\, f_1(x_i) + \varepsilon_{10i}$$

Ellipse Theory II

Solve for f1, neglecting errors

$$f_1^*(x_i) = \frac{\delta\alpha_{21}(x_i) - \delta\alpha_{20}(x_i)}{\delta C_1}$$

Repeat process, moving second bender adjustment

$$f_2^*(x_i) = \frac{\delta\alpha_{22}(x_i) - \delta\alpha_{20}(x_i)}{\delta C_2}$$

Apply regression analysis

$$\hat{\mathrm{A}} \approx \begin{bmatrix} 1 & f_1^*(x_1) & f_2^*(x_1) \\ 1 & f_1^*(x_2) & f_2^*(x_2) \\ \vdots & \vdots & \vdots \\ 1 & f_1^*(x_m) & f_2^*(x_m) \end{bmatrix}$$

Ellipse Theory III

Solve for best <u>CHANGE</u> in bender adjustments

$$\Delta\hat{C}^* = (\hat{A}'\,\hat{A})^{-1}\,\hat{A}'\,\delta\alpha_{20}$$

With dispersion

$$D(\Delta\hat{C}^*) = \sigma^2\,(\hat{A}'\,\hat{A})^{-1}$$

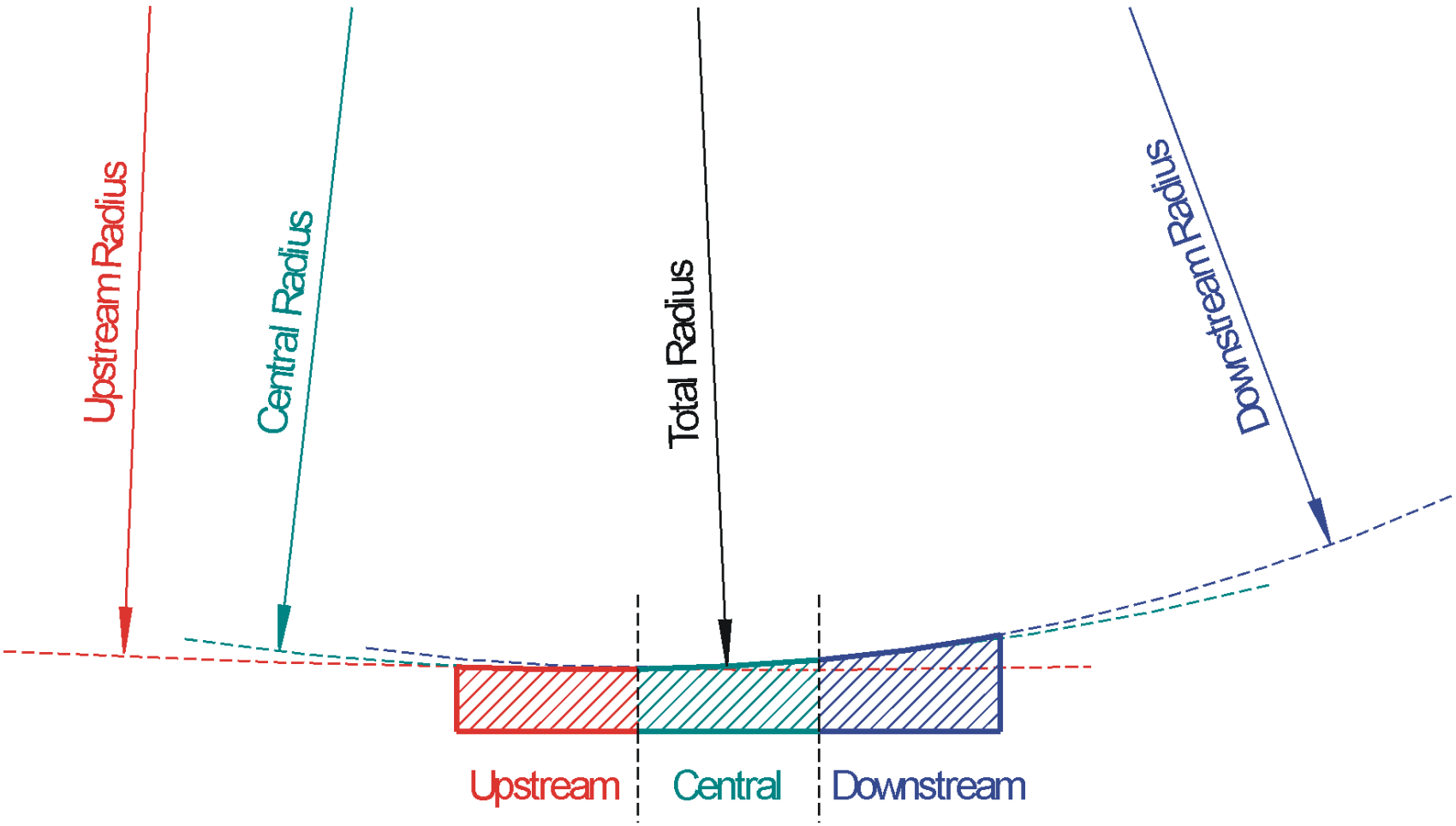
Where sigma is:

$$\sigma^2 \approx \frac{\sum_i [\delta\alpha_{20} - \Delta C_1^*\, f_1^*(x_i) - \Delta C_2^*\, f_2^*(x_i)]^2}{m-p}$$

and we differ from the reference below by C_0 and using the LTP, not beamline measurements

O. Hignette, A. Freund and E. Chinchio, "Incoherent X-ray mirror surface metrology," in *Materials, Manufacturing, and Measurement for Synchrotron Radiation Mirrors. San Diego, CA* (1997).

Now average over the previous method—compute 4 radii

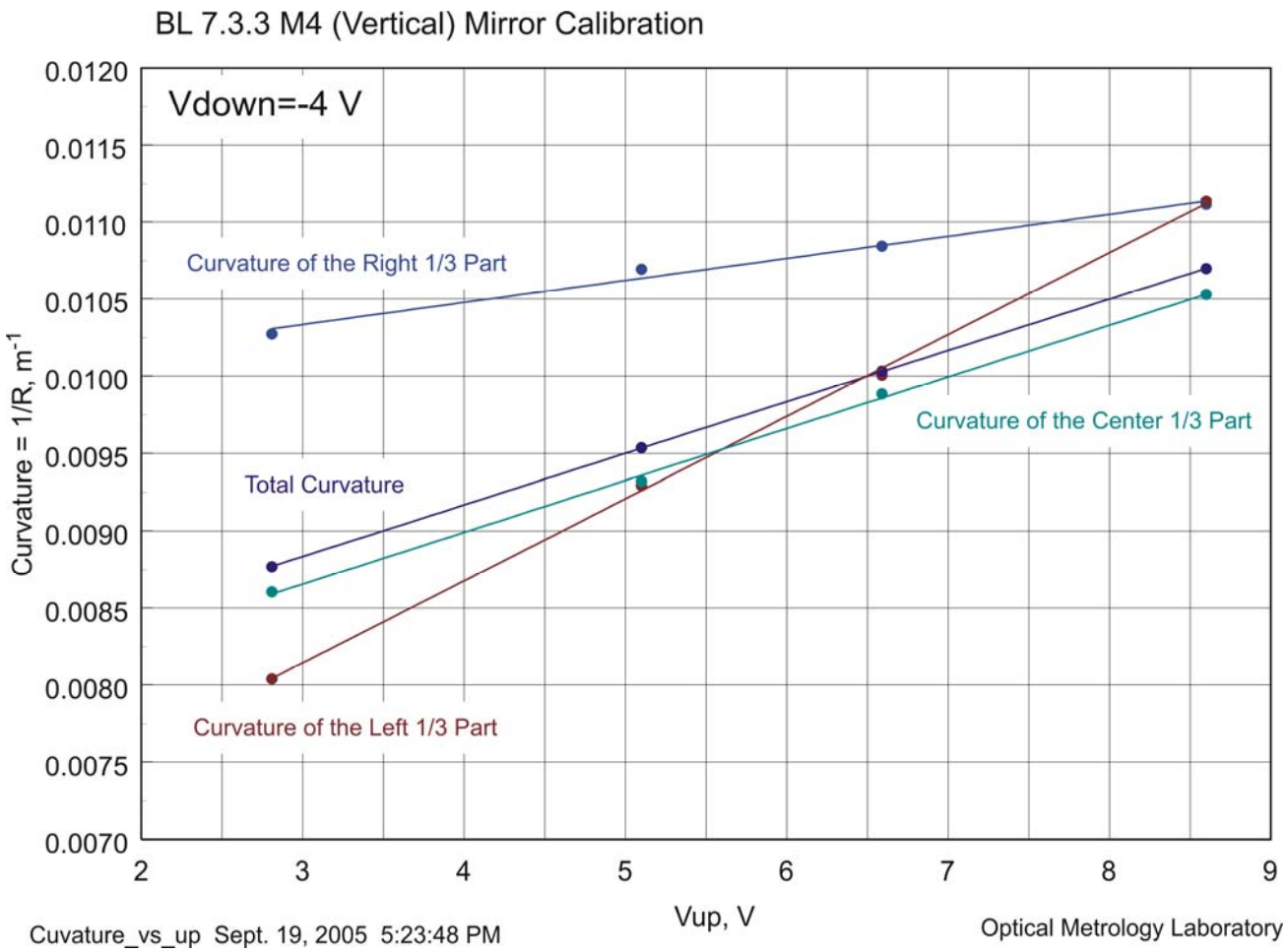


Why do this?

Left hand adjuster affects left curvature

Right hand adjuster affects right curvature

Center curvature and total curvature monitor bending

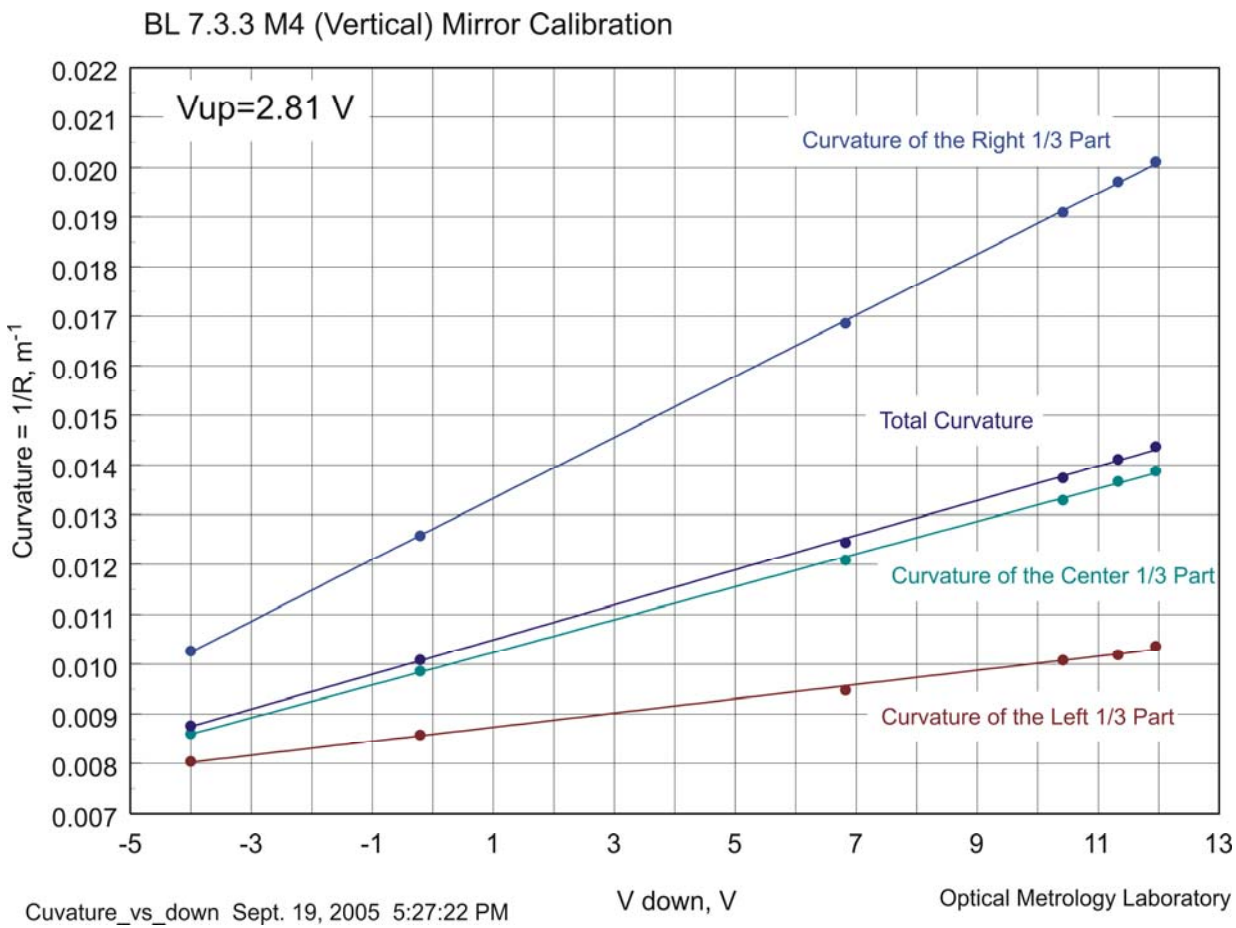


Why do this? II

Left hand adjuster affects left curvature

Right hand adjuster affects right curvature

Center curvature and total curvature monitor bending



4 Radii details

- Fit noisy LTP data to 5th order polynomial, a compromise between low pass filtering, and available adjustments

- Integrate this fit term by term, which spreads errors out, and is better than using Simpson's rule, for example

- Divide into three regions; left, center, and right

- Do regression fit exactly like the preceding analysis, except we are averaging the curvatures over many points, instead of computing the curvatures for every LTP data point.

4 Radii math I

Trick from literature, use sum of differences of areas of circles

$$Q = \sum_i \pi \left[\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2 - R^2 \right]^2$$

differentiate

$$\frac{\partial Q}{\partial R} = -4\pi R \sum_i \left[\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2 - R^2 \right] = 0$$

Divide out

$$\sum_i \left[\left(x_i - x_0\right)^2 + \left(y_i - y_0\right)^2 - R^2 \right] = 0$$

Expand

$$\sum_i \left[x_0^{\,2} + y_0^{\,2} - R^2 + x_i^{\,2} + y_i^{\,2} - 2\left(x_i x_0\right) - 2\left(y_i y_0\right) \right] = 0$$

Define variables

$$a = -2x_0, \quad b = -2y_0, \quad and\ c = x_0^{\,2} + y_0^{\,2} - R^2$$

Recast

$$\sum_i \left[c + x_i^{\,2} + y_i^{\,2} + a x_i + b y_i \right] = 0$$

4 Radii math II

Write in matrix form:

(Solve with IDL)

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ \vdots & \vdots & \vdots \\ x_m & y_m & 1 \end{bmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{bmatrix} -x_1^2 - y_1^2 \\ -x_2^2 - y_2^2 \\ \vdots \\ -x_m^2 - y_m^2 \end{bmatrix}$$

Go back to original variables

$$x_0 = -\frac{a}{2}, \quad y_0 = -\frac{b}{2}, \quad R = \sqrt{\left(\left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2 - c\right)}$$

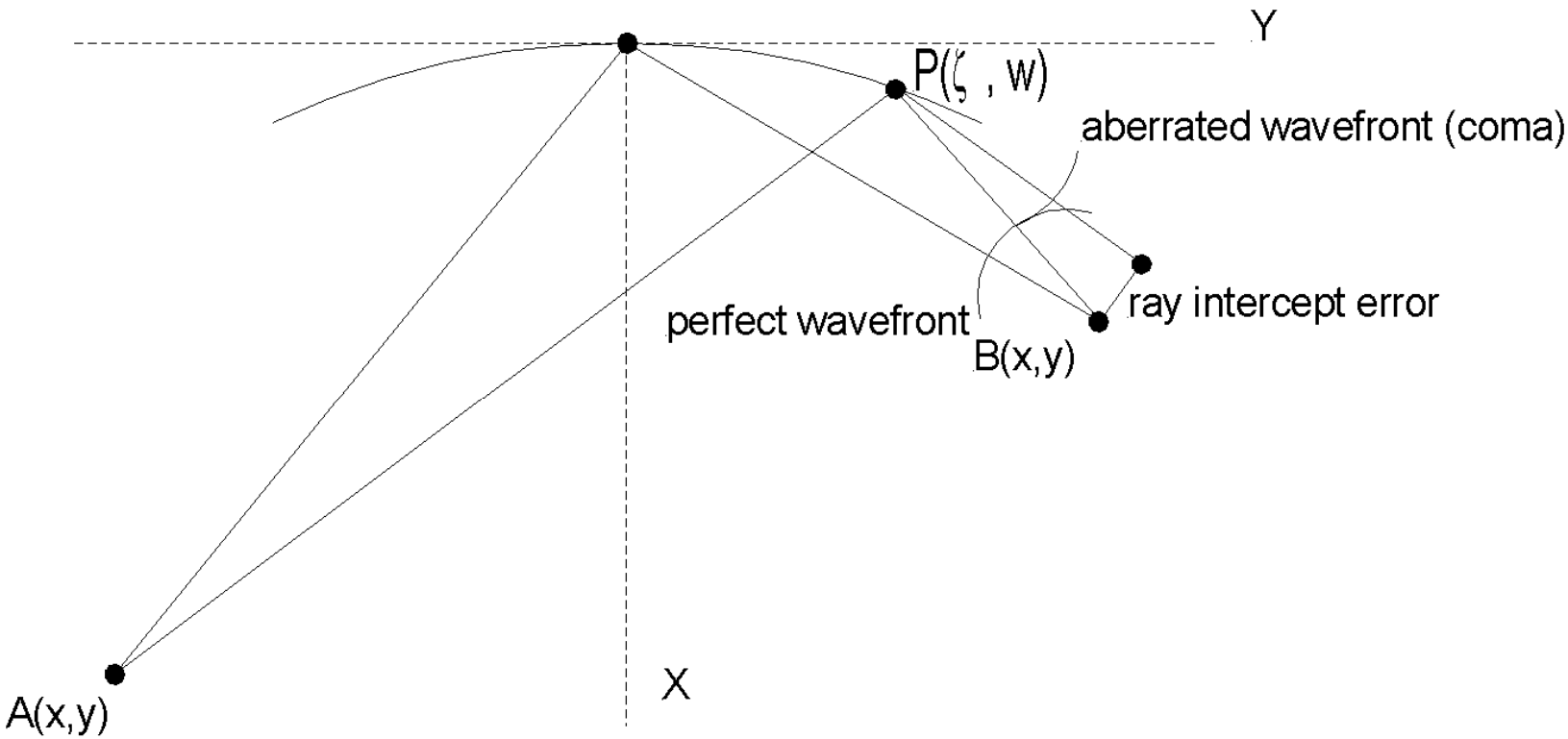
Retrieve dispersion

$$D(a,b,c) = \sigma_{IDL}{}^2 \, (\hat{A}' \, \hat{A})^{-1}$$

Figure complicated, but straightforward propagation of error

$$\sigma_R = \frac{1}{4}\sqrt{\frac{\left(\sigma_a^2 a^2 + \sigma_b^2 b^2 + 4\sigma_c^2\right)}{(R)^2}}$$

As an overall check, we use a wave front ray trace



Wave front ray trace details

Optical path is distance

$$F = \langle AP \rangle + \langle PB \rangle = \sqrt{(x - \xi)^2 + (y - w)^2}$$

where:

$$x = r\ cos\alpha \quad and \quad y = r\ cos\beta$$

Make Mac Laurin series

$$\langle AP \rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \left\{ w \frac{\partial}{\partial w} \right\}^n \langle AP \rangle \Big|_0 \equiv \sum_i C_i w^i$$

Repeat for other side

$$F_j = C_j + D_j$$

Compute ray intercept errors

$$\Delta y' = \frac{r'}{cos(\beta)} \frac{\partial (F_i l^i)}{\partial l} = \frac{i\ r'}{cos(\beta)} F_i l^{i-1}$$

Now Let's adjust

- We will MONITOR with the 4-radii method, but

- ADJUST by the full regression method based on each point of the LTP data

4-radii regression-based 1st measurement of a mirror

	original r' $\neq$ 3 m	ellipse r' = 3 m	after 1 adjustment
R_{total}	211.0 +/- 0.10 m	102.8 +/- 0.05 m	104.6 +/- 0.18 m
R_{left}	220.9 +/- 0.26 m	105.7 +/- 0.03 m	117.0 +/- 0.19 m
R_{center}	209.6 +/- 0.05 m	102.9 +/- 0.03 m	104.6 +/- 0.11 m
R_{right}	210.6 +/- 0.11 m	100.0 +/- 0.03 m	96.6 +/- 0.06 m

Summary of the data taken for the 1st adjustment

	upstream encoder	downstream encoder
original	105.825	126.294
add 10 up	115.825	126.294
add 10 down	105.825	136.294
δC_1, δC_2 calculated	**+83.448**	**+152.125**
new settings	189.273	278.419

4-radii regression-based 2nd measurement of a mirror

	original 1st adj	ellipse r' =3 m	after 2nd adjustment
R_{total}	104.6 +/- 0.18	102.8 +/- 0.05 m	102.49 +/- 0.06
R_{left}	117.0 +/- 0.19	105.7 +/- 0.03 m	107.00 +/- 0.08
R_{center}	104.6 +/- 0.11	102.9 +/- 0.03 m	102.28 +/- 0.04
R_{right}	96.6 +/- 0.06	100.0 +/- 0.03 m	100.12 +/- 0.01

Summary of the data taken for the 2nd adjustment

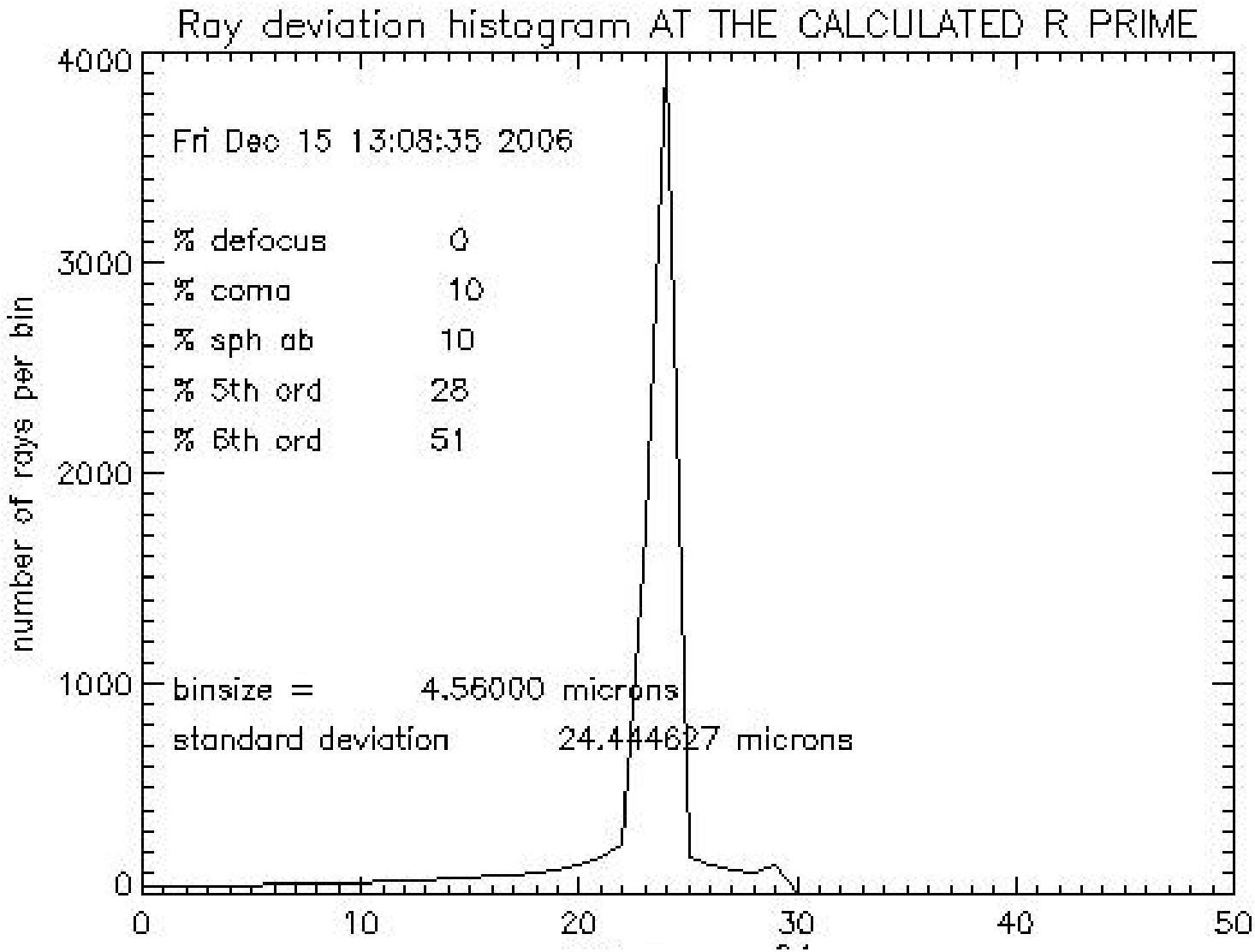
	upstream encoder	downstream encoder
original	189.273	278.419
add 10 up	199.273	278.419
add 10 down	189.273	288.419
δC_1, δC_2 calculated	**+28.390**	**-20.557**
new settings	217.663	257.862

We are there!

One more iteration gave delta values of $\delta C1$ = +1.035 and $\delta C2$= -1.154.

These small numbers confirm that we have reached the best possible adjustment with this method

Now a final check by wave front ray trace



Conclusions

- Basic linearity of two couple bending demonstrated.

- Two methods given explicitly which are approximately 10 times faster than fitting ellipses in the lab.

- Method gives complete calibration curve for the bender allowing changes in conjugate distances
- without removal from the beamline.

Observations for the future

- It is our observation that ad hoc adjustments to bendable mirrors at the beam may not be indicated, and can often ruin the careful adjustment of the mirror accomplished in metrology.

- The next step: more tightly control r, r' and θ so that the metrology/adjustment process may be completely integrated

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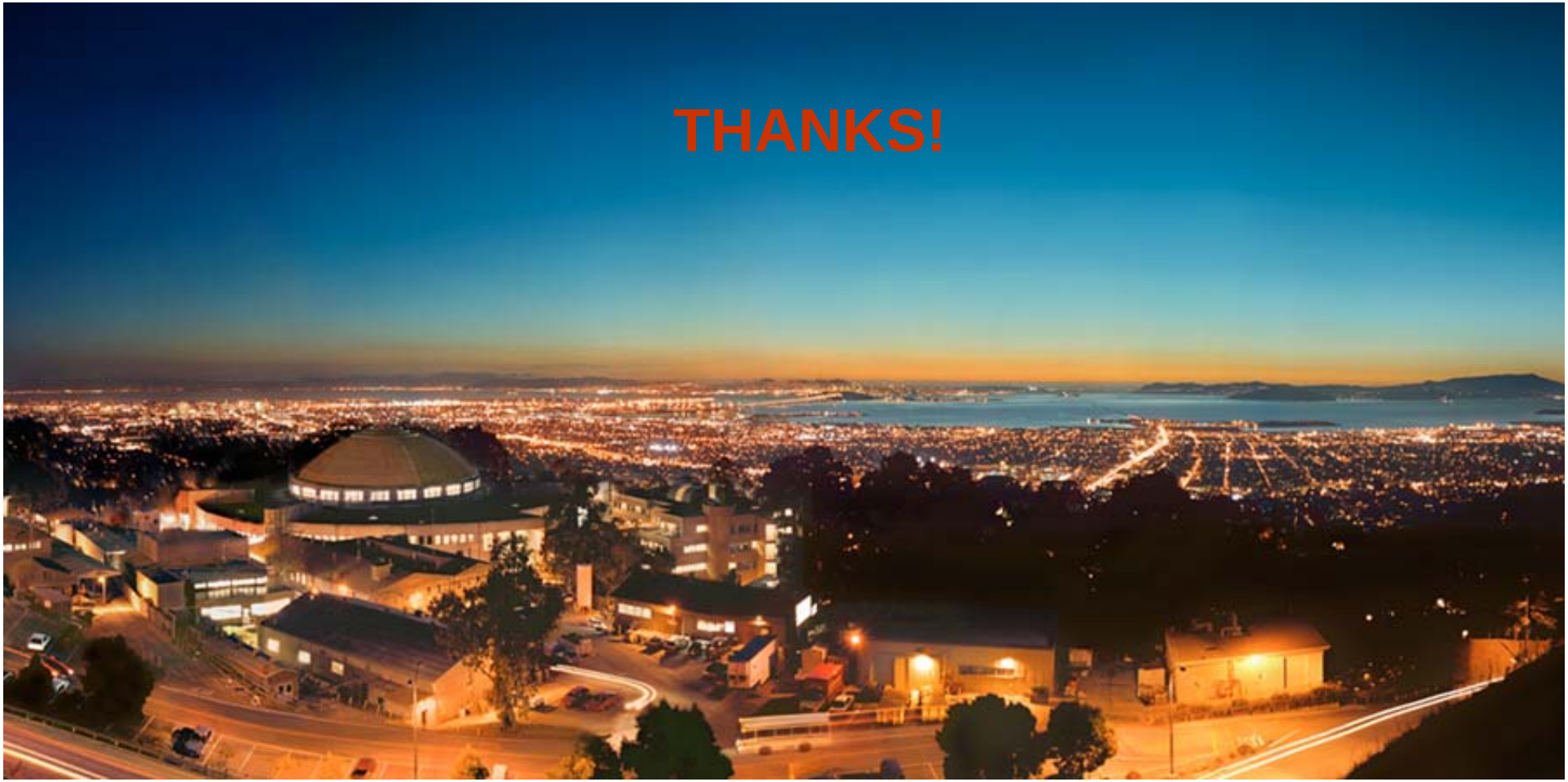

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